Exercise 8.1 (3 points) *Dimensional regularisation*

a) Simplify the following Dirac chains: $\gamma^\mu \not{a} \gamma_\mu$, $\gamma^\mu \not{a} \not{b} \gamma_\mu$, $\gamma^\mu \not{a} \not{b} \not{c} \gamma_\mu$.

b) Show that $\int d^D q \, \left(q^2\right)^\alpha = 0$ for all complex variables D and α.

c) Calculate the tensor integral

$$I_{n,\mu\nu}(A) \;=\; \int d^D q \, \frac{q_\mu q_\nu}{(q^2 - A + i0)^n}$$

by applying the ansatz $I_{n,\mu\nu}(A) = g_{\mu\nu} \, J_n(A)$ and reducing the integral to the scalar integral

$$I_n(A) \;=\; \int d^D q \, \frac{1}{(q^2 - A + i0)^n} \; .$$

Exercise 8.2 (4 points) *Scalar two-point integrals*

The scalar two-point function is given by

$$\begin{aligned} B_0(p^2, m_0, m_1) \;&=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \, \frac{1}{(q^2 - m_0^2 + i0)\,[(q+p)^2 - m_1^2 + i0]} \\ &=\; \Delta - \int_0^1 dx \, \ln\left[\frac{x^2 p^2 - x(p^2 - m_1^2 + m_0^2) + m_0^2 - i0}{\mu^2}\right] \;+\; \mathcal{O}(D-4), \end{aligned}$$

$$\text{where} \quad \Delta = \frac{2}{4-D} - \gamma_E + \ln(4\pi).$$

Calculate $B_0(p^2, m_0, m_1)$ for the following special cases:

a) $B_0(p^2, m, 0)$,

b) $B_0(p^2, m, m)$,

[Hint: First assume $p^2 < 0$, so that B_0 becomes a purely real integral, and then derive the final result by analytic continuation to arbitrary real p^2.]

Please turn over!

Exercise 8.3 (3 points) *Two-point tensor integral*

The two-point tensor integral of rank 2 is given by

$$B_{\mu\nu}(p, m_0, m_1) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \, \frac{q_\mu q_\nu}{(q^2 - m_0^2 + i0)\,[(q+p)^2 - m_1^2 + i0]}$$

and can be rewritten, applying the covariant decomposition, as follows

$$B_{\mu\nu}(p, m_0, m_1) \;=\; g_{\mu\nu} B_{00}(p^2, m_0, m_1) + p_\mu p_\nu B_{11}(p^2, m_0, m_1).$$

a) Express the tensor coefficients B_{00} and B_{11} in terms of the scalar integrals A_0 and B_0,

$$A_0(m) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \, \frac{1}{q^2 - m^2 + i0},$$

$$B_0(p^2, m_0, m_1) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \, \frac{1}{(q^2 - m_0^2 + i0)\,[(q+p)^2 - m_1^2 + i0]}.$$

b) Determine the UV-divergent part of $B_{\mu\nu}$ and calculate $(D-4)B_{\mu\nu}$ to the order of $\mathcal{O}(D-4)$.