

# Resonant Top Quarks at Hadron Colliders

Andrew Papanastasiou\*



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\*In collaboration with:

Pietro Falgari (Utrecht), Adrian Signer (PSI)

and

Rikkert Frederix (CERN), Stefano Frixione (CERN/EPFL),

Valentin Hirschi (EPFL), Fabio Maltoni (Louvain)

# Outline

Unstable particle treatments

Effective theory approach

Results

Conclusions/Outlook

## Why tops?

Heaviest member of the Standard Model and millions produced at LHC:

- top parameters/properties important in precision checks of SM
- tops contribute to backgrounds to other processes of interest, e.g.  $h \rightarrow W^+ W^-$
- sensitive to many BSM scenarios

### Why off-shell tops ?

- mis-estimation of contribution of tops to backgrounds, when e.g. on-shell assumption made
- tagging top quarks
- template fitting (e.g. for measuring  $m_t$ ):  
on-shell & stable assumptions often made (at the matrix element level).

Are systematics related to ignoring offshellness of top (and all this entails) understood and controlled?

## This talk

The focus in this talk: production and decay of tops

- a method that allows for the inclusion of off-shell effects
- comparison between methods for treating production and decay of tops

Discussion will be at the parton/hard-scattering level, i.e. no parton-shower or hadronization effects.

Stable top not discussed, despite the huge progress at both NLO ( $t\bar{t} + X$ ) and NNLO ( $t\bar{t}$ ).

## Heavy Unstable Particles: on-shell / NWA

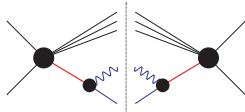
$$p_X^2 = M_X^2$$

Decay the unstable particle via the NWA

✓ Pros:

- include corrections to production & decay
- good approx. for many observables

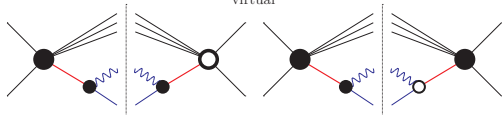
Born



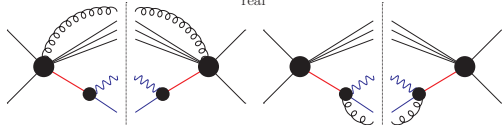
☹ Not included:

- non-factorizable corrections
- off-shell/finite-width effects in NLO amplitudes
- non-resonant effects

virtual



real



$t\bar{t}$ : [Bernreuther et al. '01, '04 ; Melnikov Schulze '09 ; Campbell, Ellis '12]

single-top: [Campbell et al. '04; Cao et al. '09, '10]

## Heavy Unstable Particles: on-shell / NWA

$$p_X^2 = M_X^2$$

NWA:

$$\frac{1}{(p_X^2 - m_X^2)^2 + m_X^2 \Gamma_X^2} \rightarrow \frac{\pi}{m_X \Gamma_X} \delta(p_X^2 - m_X^2) + \mathcal{O}(\Gamma_X/m_X)$$

For processes involving top quarks, this is  $\sim 1\%$

For the inclusive production of an unstable particle, non-factorizable corrections are parametrically suppressed  $\mathcal{O}(\Gamma_X/M_X)$

[theorems of: Fadin, Khoze, Martin '94 ; Melnikov, Yakovlev '94]

Is this still the case for more exclusive observables (e.g.  $M(W^+, J_b)$ ) and at the fully differential level?

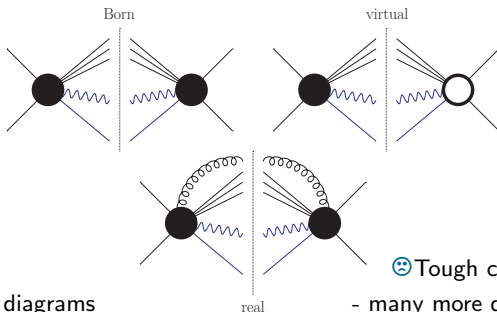
More generally, one would like to understand at the differential level:

- what changes in the resonant contributions when the on-shell assumption is relaxed?
- where do non-resonant/background contributions play important roles?

# Heavy Unstable Particles: off-shell / CMS

$$p_X^2 \neq M_X^2$$

Much more than just the resonant contributions included in the NWA



✓ Included:

- background diagrams
- non-factorizable corrections
- resonant/non-resonant interferences
- off-shell/finite-width effects in NLO amplitudes
- valid in all regions of phase-space

☹ Tough calculation:

- many more diagrams
- many multi-point loop diagrams with complex masses

## Heavy Unstable Particles: off-shell / CMS

$$p_X^2 \neq M_X^2$$

Finite-width in propagator arises from the resummation of a particular class of higher-order correction.

Since this operation mixes up orders in perturbation theory, if it is not done with care, gauge-invariance can be broken (even at LO).

The complex-mass scheme (CMS) [Denner, Dittmaier, Roth, Wieders '05] is a generalization of the on-shell renormalization scheme:

$$m_{t,0} = \mu_t + \delta m_t, \quad \mu_t^2 = m_t^2 - im_t \Gamma_t$$

→  $\delta m_t$  fixed so  $\mu_t^2$  corresponds to complex pole of propagator

→ complex masses introduced consistently at the level of the Lagrangian.

$W^+ b W^- \bar{b}$ : [Denner, Dittmaier, Kallweit, Pozzorini '10 & '12]

[Bevilacqua, Czakon, van Hameren, Papadopoulos '10]

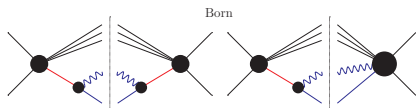
$W^+ bj$ : [Pittau '96 (s-ch, final-state non-factorizable corrections)]

[Frederix, Frixione, Hirschi, Maltoni, AP - to appear (t-ch)]

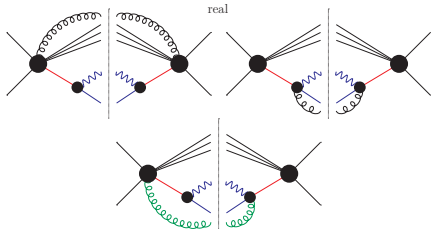
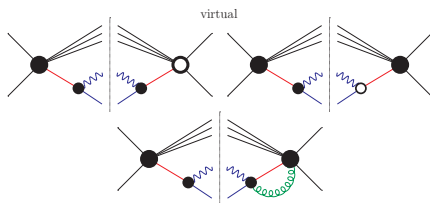


# Heavy Unstable Particles: resonant / ET

$$p_X^2 \sim M_X^2$$



systematically pick out leading contributions in this region



## ✓ Pros:

- dominant offshell/width effects
- non-factorizable corrections
- relevant non-resonant effects kept
- easy to pick out relative importance of different top contributions

## ☹ Disadvantages:

- not valid over full phase-space
- not a computation of the full process

**Single-top:** [Falgari,Mellor,Signer '10; Falgari,Gianuzzi,Mellor,Signer '11]

**$t\bar{t}$ :** [Falgari, AP, Signer '13 ( $q\bar{q}$  channel)]

## Scales in Processes with Tops

Top production and decay processes separated in space/time by  $\sim 1/\Gamma_t$

Large(ish)  $\Gamma_t \Rightarrow$  production/decay connections can only really happen when emitted gluons induce long-range interactions

$\Rightarrow$  dominant contributions from interconnections come from regions where gluons are **soft**,  $p_g \sim \Gamma_t$

( Hard gluons  $p_g \sim m_t$  induce short range interactions )

**Idea:** pick out dominant/relevant contributions

$\rightarrow$  use the widely separated physical scales present to simplify calculation

$\Gamma_t \ll m_t \rightarrow$  expand full amplitude in  $\frac{\Gamma_t}{m_t} \ll 1$

When top is resonant:  $\frac{\Delta_t}{m_t^2} := \frac{p_t^2 - \mu_t^2}{m_t^2} \sim \frac{\Gamma_t m_t}{m_t^2} = \frac{\Gamma_t}{m_t}$

Expansion in  $\frac{\Gamma_t}{m_t} \leftrightarrow$  expansion in top **virtuality**

## Effective Theory (ET) expansion: summary

Combine standard expansion in  $\alpha_s$  and  $\alpha_w$  with an expansion in  $\Delta_t$

[pole expansion: A. Aeppli et. al. '94 ]

[& its systematization: A.P. Chapovsky et. al. '01 ; M. Beneke et. al. '04 ]

Introduce power-counting:  $\frac{\Delta_t}{m_t^2} \sim \alpha_w \sim \alpha_s^2 \sim \delta \ll 1$  ( $\Gamma_t \sim \alpha_w m_t$ )

→ Expand to NLO in  $\delta$ : corrections of  $\delta^{1/2}$  to the LO term in expansion.

→ corrections of  $\mathcal{O}(\delta^{1/2})$ : keep

→ corrections of  $\mathcal{O}(\delta^1)$ : safely drop

→ corrections of  $\mathcal{O}(\delta^{-1})$ : leading → resum (leads to width)

→ *only* Dyson-resum resonant propagators

✓ expansion in  $\delta$  is gauge invariant and systematically improvable

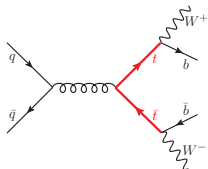
[A.P. Chapovsky et. al. '01 ; M. Beneke et. al. '04]

In resonant region,  $\delta$ -expansion re-arranges full perturbation theory.

→ provides an intuitive picture of the structure of corrections

## ET expansion: Born

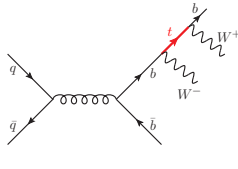
Power-counting for tree-level diagrams:



$$\sim \alpha_s \alpha_w A_{(-2)}^{(1,1)}$$

$$\sim \alpha_s \alpha_w \frac{1}{\Delta_t \Delta_{\bar{t}}}$$

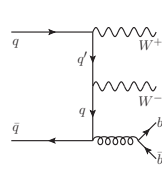
$$\sim \delta^{1/2} \delta / \delta^2 = \delta^{-1/2}$$



$$\sim \alpha_s \alpha_w A_{(-1)}^{(1,1)}$$

$$\sim \alpha_s \alpha_w \frac{1}{\Delta_t}$$

$$\sim \delta^{1/2} \delta / \delta = \delta^{1/2}$$



$$\sim \alpha_s \alpha_w A_{(0)}^{(1,1)}$$

$$\sim \alpha_s \alpha_w$$

$$\sim \delta^{1/2} \delta = \delta^{3/2}$$

Note:  $\delta$ -scalings here are how the *leading* contributions of an expansion of these diagrams would scale.

## ET expansion: Born

Colour summed/averaged tree-level matrix element for full process can be written as:

$$\begin{aligned}
 \mathcal{M}_{\text{full}}^{\text{tree}} &= \frac{C_F}{2N_c} \alpha_s^2 \alpha_w^2 \left[ \left| A_{(-2)}^{(1,1)} \right|^2 + 2\text{Re} \left\{ A_{(-2)}^{(1,1)} A_{(-1)}^{(1,1)*} \right\} + \dots \right] \\
 &+ \alpha_w^2 \left[ \left| A_{(-2)}^{(0,2)} \right|^2 + \dots \right] \\
 &= \sim \delta^{-1} + \sim \delta^0 + \dots
 \end{aligned}$$

Ellipses indicate terms of  $\mathcal{O}(\delta)$  and higher.

## ET expansion: higher orders

To consistently go to higher orders in the  $\delta$  expansion must include higher orders in standard coupling expansions.

Think about  $\alpha_s$ -corrections to leading double resonant contributions

$$\text{loop corrections} \sim A_{(-2)}^{(2,1)}$$

$$\mathcal{M}^{\text{Virt}} \sim \alpha_s^3 \alpha_w^2 2\text{Re} \left\{ A_{(-2)}^{(2,1)} A_{(-2)}^{(1,1)*} \right\} \sim \delta^{-\frac{1}{2}}$$

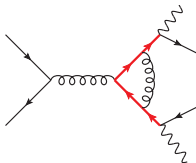
$$\text{real emission corrections} \sim A_{(-2)}^{(3/2,1)}$$

$$\mathcal{M}^{\text{Real}} \sim \alpha_s^3 \alpha_w^2 \left| A_{(-2)}^{(\frac{3}{2},1)} \right| \sim \delta^{-\frac{1}{2}}$$

Here the aim is to compute the next-to-leading term in the  $\delta$  expansion, i.e. we want to keep all contributions scaling as  $\delta^{-1/2}$  in the matrix element  $\rightarrow$  'NLO'.

## ET expansion: loop example

Method of regions [M. Beneke, V. A. Smirnov '98] used to extract pieces of loop diagrams we must keep.



$$= \alpha_s^2 \alpha_w \dots \frac{1}{\Delta_t \Delta_{\bar{t}}} \int [dk] \frac{\gamma^\mu (\not{p}_t - \not{k} + m_t) \gamma_\rho (\not{p}_{\bar{t}} + \not{k} - m_t) \gamma_\mu}{k^2 ((p_t - k)^2 - m_t^2) ((p_{\bar{t}} + k)^2 - m_t^2)} \dots$$

Start from full amplitudes and expand in relevant regions...

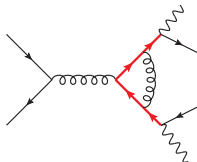
For the case at hand we must expand regions where virtual gluon is:

- hard,  $k_0 \sim \vec{k} \sim m_t$
- soft,  $k_0 \sim \vec{k} \sim m_t \delta$

Drop factors of  $m_t$  to ease discussion.

## ET expansion: loop example - hard

In hard region, virtual gluon momentum scales as  $k_0 \sim \vec{k} \sim 1$ .



$$\sim \delta \cdot \delta \cdot \frac{1}{\delta \cdot \delta} \cdot 1 \cdot \frac{1}{1 \cdot 1 \cdot 1} \sim 1 \rightarrow \text{keep!}$$

$$= \alpha_s^2 \alpha_w \dots \frac{1}{\Delta_t \Delta_{\bar{t}}} \int [dk] \frac{\gamma^\mu (\not{p}_t - \not{k} + m_t) \gamma_\rho (\not{p}_{\bar{t}} + \not{k} - m_{\bar{t}}) \gamma_\mu}{k^2 ((p_t - k)^2 - m_t^2) ((p_{\bar{t}} + k)^2 - m_{\bar{t}}^2)} \dots$$

$$\text{hard: } (p_t - k)^2 - m_t^2 = k^2 - 2k \cdot p_t + D_t \rightarrow k^2 - 2k \cdot p_t.$$

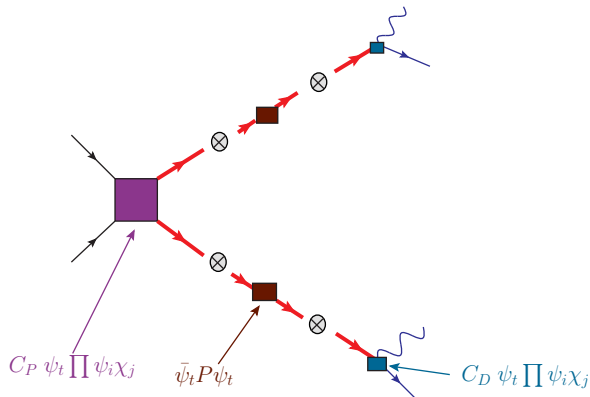
$$(D_t = p_t^2 - m_t^2 \sim \delta)$$

$$\rightarrow \alpha_s^2 \alpha_w \dots \frac{1}{\Delta_t \Delta_{\bar{t}}} \int [dk] \frac{\gamma^\mu (\not{p}_t - \not{k} + m_t) \gamma_\rho (\not{p}_{\bar{t}} + \not{k} - m_{\bar{t}}) \gamma_\mu}{k^2 (k^2 - 2k \cdot p_t) (k^2 + 2k \cdot p_{\bar{t}})} \dots$$



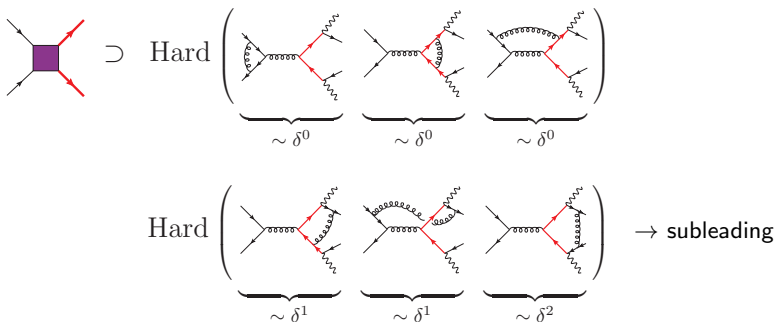
## ET structure: hard corrections / wilson coefficients

“Hard” corrections: contained in matching coefficients multiplying  
**production**, **propagation**, **decay** operators



## ET structure: hard corrections / wilson coefficients

Example:  $t\bar{t}$  production operator:



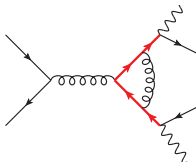
Expansion in  $\delta$  of matrix elements, requires the expansion of the external momenta around  $p_{t,\bar{t}}^2 = \mu_{t,\bar{t}}^2 = m_t^2 + \mathcal{O}(\delta)$

$\leftrightarrow$  analogous to formal effective theory, where matching performed 'on-shell'

Project off-shell momenta onto on-shell configurations to evaluate matrix elements.

## ET expansion: loop example - soft

In soft region, virtual gluon momentum scales as  $k_0 \sim \vec{k} \sim \delta$ .



$$\sim \delta \cdot \delta \cdot \frac{1}{\delta \cdot \delta} \cdot \delta^4 \cdot \frac{1}{\delta^2 \cdot \delta \cdot \delta} \sim 1 \rightarrow \text{keep!}$$

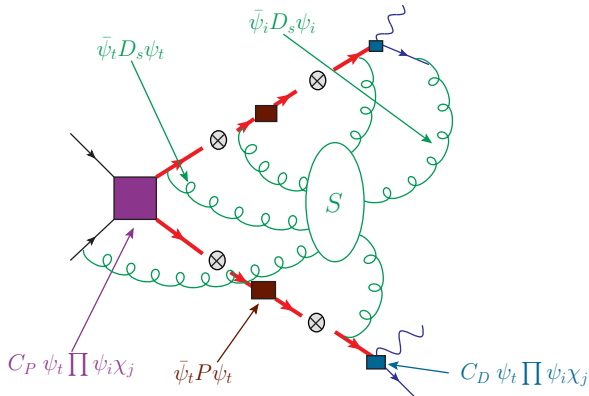
$$= \alpha_s^2 \alpha_w \dots \frac{1}{\Delta_t \Delta_{\bar{t}}} \int [dk] \frac{\gamma^\mu (\not{p}_t - \not{k} + m_t) \gamma_\rho (\not{p}_{\bar{t}} + \not{k} - m_{\bar{t}}) \gamma_\mu}{k^2 ((p_t - k)^2 - m_t^2) ((p_{\bar{t}} + k)^2 - m_{\bar{t}}^2)} \dots$$

$$\text{soft: } (p_t - k)^2 - m_t^2 = k^2 - 2k \cdot p_t + D_t \rightarrow -2k \cdot p_t + \Delta_t.$$

$$\rightarrow \alpha_s^2 \alpha_w \dots \frac{1}{\Delta_t \Delta_{\bar{t}}} \int [dk] \frac{\gamma^\mu (\not{p}_t + m_t) \gamma_\rho (\not{p}_{\bar{t}} - m_{\bar{t}}) \gamma_\mu}{k^2 (-2k \cdot p_t + \Delta_t) (2k \cdot p_{\bar{t}} + \Delta_{\bar{t}})} \dots$$

## ET structure: soft corrections / dynamical d.o.f.

“Soft” corrections: **dynamical** degrees of freedom left in theory after hard (high virtuality) modes integrated out.



→ separation of contributions which live at different scales  $\mu_h$  and  $\mu_s$

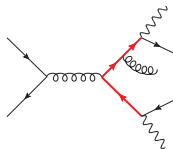
## Real contributions

Want to study **exclusive** observables, so must include real emission diagrams

→ do this in a way consistent with Method of Regions used for loops

However, harder to identify what the correct expansion parameter is.

Consider a gluon emitted (with momentum  $k$ ) off a top quark line:



In resonant region possibilities are:

- $p_t^2 \sim m_t^2$
- $(p_t + k)^2 \sim m_t^2$
- both

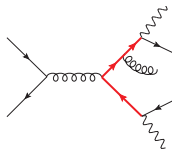
## Real contributions

Want to study **exclusive** observables, so must include real emission diagrams

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However, harder to identify what the correct expansion parameter is.

Consider a gluon emitted (with momentum  $k$ ) off a top quark line:



In resonant region possibilities are:

- $p_t^2 \sim m_t^2$  →  $\delta \sim (p_t^2 - m_t^2) ?$
- $(p_t + k)^2 \sim m_t^2$  →  $\delta \sim ((p_t + k)^2 - m_t^2) ?$
- both

## Real contributions

Previously, what we had to do was modify standard subtraction method:

$$d\sigma^{NLO} = \left( d\sigma^V + \int_1 d\sigma^{R,c.t.} \right) + \int_1 (d\sigma^R - d\sigma^{R,c.t.})$$

by

$$d\sigma^{NLO} \simeq \left( d\sigma_{\text{exp}}^V + \int_1 d\sigma_{\text{exp}}^{R,c.t.} \right) + \int_1 (d\sigma^R - d\sigma^{R,c.t.})$$

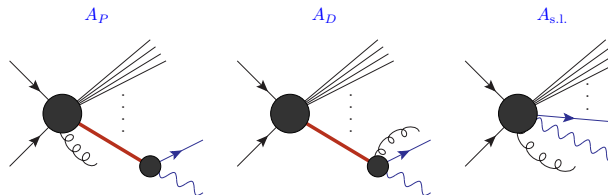
Add back something different from what you subtract - but difference suppressed in  $\delta$  ... unsatisfactory however.

For ambiguous contributions use of the identity:

$$\frac{1}{(p_t^2 - m_t^2) ((p_t + k)^2 - m_t^2)} = \frac{1}{2p_t \cdot k} \left[ \frac{1}{(p_t^2 - m_t^2)} - \frac{1}{((p_t + k)^2 - m_t^2)} \right],$$

which helps make the separation.

## Separation of real contributions



$A_P \sim \frac{1}{\Delta_t}$  and approximates full real amplitude when  $(p_W + p_b)^2 \sim m_t^2$

$A_D \sim \frac{1}{\Delta_{tg}}$  and approximates full real amplitude when  $(p_W + p_b + p_g)^2 \sim m_t^2$

$\rightarrow A_P + A_D + A_{s.l.}$  approximates full amplitude **always**

$\int d\Phi_g |A_{P/D}|^2$  results in  $\frac{1}{\epsilon^2}$  and  $\frac{1}{\epsilon}$  poles  $\rightarrow$  cancelled by **hard virtual** pieces.

$\int d\Phi_g 2 \operatorname{Re}(A_P [A_D]^*)$  results in a standard  $\frac{1}{\epsilon}$  soft-pole

$\rightarrow$  cancelled by  $\frac{1}{\epsilon}$ -pole of the **soft virtual** contribution.



## Separation of contributions

We note that the non-factorizable term

$$2\text{Re}(A_P [A_D]^*) \sim \frac{1}{\Delta_t \Delta_{tg}}$$

which gives a sizeable contribution *only* if **both**  $\Delta_t$  and  $\Delta_{tg}$  are resonant, i.e. when the emitted gluon is soft

→ confirms that, as for virtual correction, the non-factorizable real corrections are encoded by soft radiation.

Can now write:

$$d\sigma^{NLO} = d\sigma^P + d\sigma^{D, t} + d\sigma^{D, \bar{t}} + d\sigma^{NF}$$

and study each of these contributions separately in a consistent manner.

→ in particular, can evaluate soft corrections at their 'natural' scale,  $\mu_s \simeq m_t \delta$ .

## Some comments

- **dependence** on projection used  
(however, difference between different projections is at **an order higher** than order which we aim for)
- pair-production near threshold: expansion in  $\delta$  no longer captures the relevant physics (coulomb interactions  $\rightarrow$  would have to match to a different expansion). 😞
- with ET method we study  $t\bar{t}$  above threshold  
 $\rightarrow$  can be done via a physical cut on final states.

## Resonant $t\bar{t}$ production: process definition

Collider : 1.96 TeV Tevatron

$W$ -decays to leptons via NWA

Use MSTW2008NLO PDF set, with  $\alpha_s(M_Z) = 0.12018$ ,

$$m_t^{\text{pole}} = 172.9 \text{ GeV} \quad M_W = 80.4 \text{ GeV}$$

$$\Gamma_t = 1.366 \text{ GeV} \quad \Gamma_W = 2.140 \text{ GeV}$$

$$\mu_F = \mu_R = m_t \quad \mu_s = m_t \delta, \delta = 0.02$$

Cuts:  $k_{\perp}$ -algorithm,  $R_{\text{jet}} = 0.7$

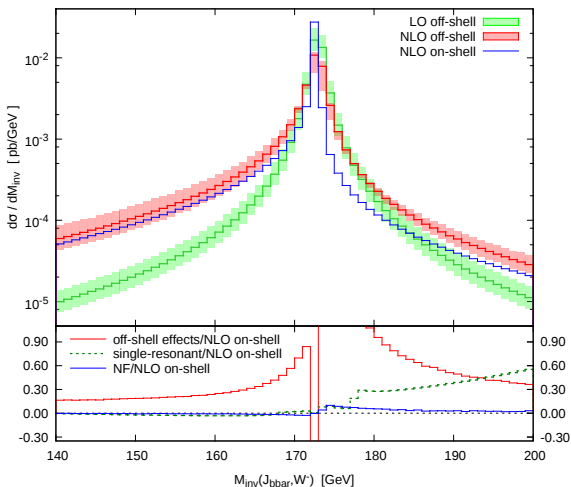
$$p_T(J_b), p_T(J_{\bar{b}}) > 15 \text{ GeV}$$

$$p_T(l^+), p_T(l^-) > 15 \text{ GeV} \quad \cancel{E}_T > 20 \text{ GeV}$$

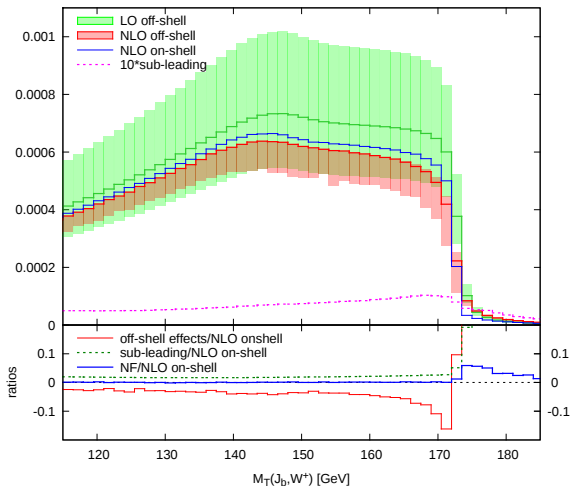
$$140 < M(W^+, J_b) < 200 \text{ GeV} \quad 140 < M(W^-, J_{\bar{b}}) < 200 \text{ GeV}$$

$$M_{t\bar{t}} > 350 \text{ GeV}$$

Note:  $M_{t\bar{t}} = M_{t\bar{t}}^{\text{rec.}} = M((W^+, J_b), (W^-, J_{\bar{b}}))$

Resonant  $t\bar{t}$  production: invariant mass

- off-shell effects large near peak
- pure non-factorizable corrections small
- sub-leading terms in  $\delta$  grow away from peak

Resonant  $t\bar{t}$  production: transverse mass

- below kinematic edge, off-shell effects  $\sim 1\text{-}2\%$
- near and beyond the edge off-shell effects important
- non-factorizable terms insignificant, except perhaps near the edge

## $t$ -channel EW $W^+ b j$ production

Study  $t$ -channel EW  $W^+ J_b J_{\text{light}}$  production in the 5-flavour scheme, using CMS, i.e.  $t$ -channel single-top with off-shell effects

[Frederix,Frixione,Hirschi,Maltoni,AP - to appear]

- in the framework of AMC@NLO [Frederix,Frixione,Hirschi,Maltoni,Pittau,Torrielli]
- CMS recently implemented in AMC@NLO

[Buarque Franzosi,Hirschi,Mattelaer - to appear]

Comparison is made at NLO to:

- NWA (as implemented in MCFM [Campbell,Ellis,Tramontano '04] ) and
- ET [Falgari,Mellor,Signer '10] (strict 'NLO' in  $\delta$ )

## $t$ -channel EW $W^+ bj$ production

Collider : 8 TeV LHC

Use MSTW2008NLO PDF set, with  $\alpha_s(M_Z) = 0.12018$ ,

$$m_t^{\text{pole}} = 173.2 \text{ GeV} \quad M_W = 80.398 \text{ GeV} \quad M_Z = 80.398 \text{ GeV}$$

$$\Gamma_t^{\text{LO}} = 1.5017 \text{ GeV} \quad \Gamma_Z = 2.4952 \text{ GeV}$$

$$\mu = \mu_F = \mu_R = m_t/2 \quad V_{tb} = 1$$

Cuts:  $k_{\perp}$ -algorithm,  $R_{\text{jet}} = 0.5$

$$p_T(J_b) > 25 \text{ GeV}$$

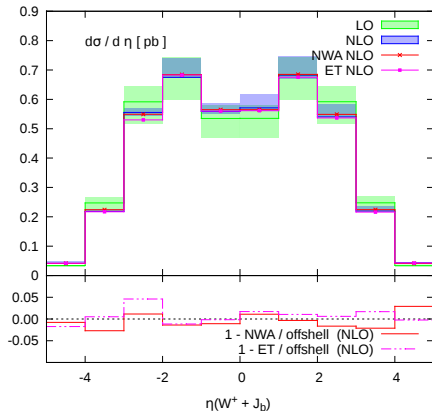
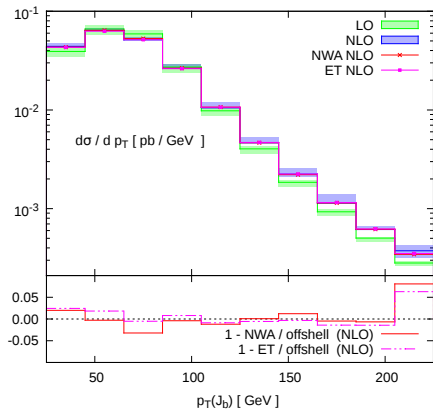
$$p_T(J_{\text{light}}) > 25 \text{ GeV}$$

$$\eta(J_b) < 4.5$$

$$\eta(J_{\text{light}}) < 4.5$$

$$140 < M(W^+, J_b) < 200 \text{ GeV}$$

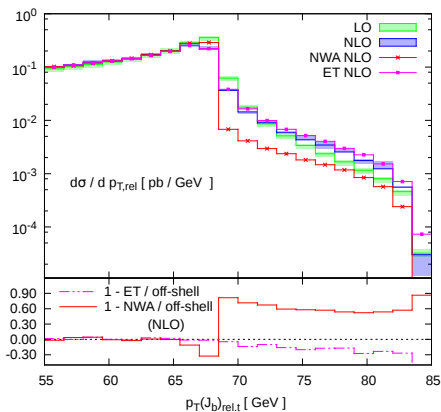
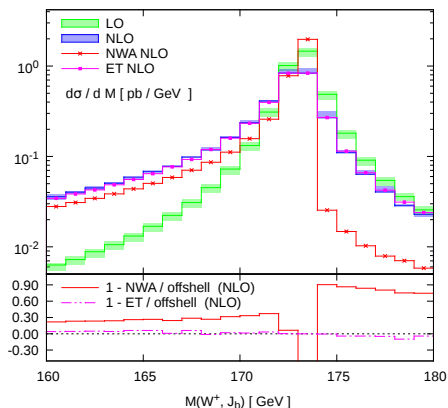
## $t$ -channel EW $W^+ b j$ production: inclusive observables



Differences between NWA, ET and fully offshell (CMS) approaches are small.



## $t$ -channel EW $W^+ b j$ production: less inclusive observables



Significant differences in shapes of distributions between NWA and ET & CMS approaches.

In RHS plot, terms subleading in  $\delta$  i.e. resonant/non-resonant interferences and subleading- $\Gamma_t$  effects become noticeable, as expected.

## Conclusions

- discussed an effective theory approach to unstable particle production
- expansion valid in the resonant region
- application to  $t\bar{t}$  and single-top
- comparison to NWA and complex-mass scheme in single-top
  - very good agreement between CMS and ET in resonant region
  - differences between CMS and ET grow away from resonant region (as we expect)
- off-shell effects small in general (NWA a v. good approximation)
- ... but can be large for observables sensitive to  $M(W^+, J_b)$
- resonant/non-resonant interferences increase in importance away from resonance
- expect larger differences if final-state cuts are such that non-resonant contributions are enhanced

## Outlook and further work

In progress:

- $t\bar{t}$ :  $gg$  &  $qg$  channels (no conceptual problems now, 'just' a matter of implementation)
- approximate NNLO in production (with A.Signer and A.Broggio) in fashion of [Ahrens,Ferrogli,Neubert,Pecjak & Yang] (...still early days)

Future:

- addition of PS to unstable single-top (?)

Thanks for listening  
and thank you for the invitation to visit Freiburg!