Econometric Risk Management in Finance
Session 5

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Main Categories of Risk

- **Market**
  - Loss due to adverse changes in the market

- **Business/Financial**
  - Risk affecting the business in terms of financial development and growth

- **Operational**
  - Management failure, system software failure, human error etc.

- **Compliance**
  - Requirements to do in order to comply with the law and regulations
Managing Linear Risk

- Hedging
  An investment made in order to reduce the risk of adverse price movements in another investment, by taking an offsetting position in a related security, such as an option, a short sale or an “inverse fund”.

(http://www.confidentstrategies.com/glossary.htm)
Futures market, Forward and Future Contracts

- **Futures market** participants buy and sell standardized future contracts according to the interaction of the competing expectations of both parts.

- **Forward contract**, the seller of a commodity, financial instrument or equity agrees to deliver to the buyer a specified amount of the products at an agreed price at a specified date in the future.

Forward contracts are privately negotiated between two parties and unless the other party agrees, not obeying a forward contract is very difficult.

- **Option**, in which the parties have a choice of not to exercise the option. An option is a contract that gives the investor the right, but not an obligation, to buy or sell a commodity/product at a specified price within a specified time period.

- **Traders and brokers** trade the contracts. who shout the bids and offers on organized exchanges in a variety of commodities are named as futures contracts. Futures contracts are more useful for risk management and hedging, because they allow parties discover prices before the commodities are traded and also the specifications are clearly defined in a futures contract.
Two types of traders: hedgers and speculators. **Hedgers** try to reduce the price risk or establish prices for commodities by using adverse price changes. **Speculators** try to make profit by anticipating the price changes of a commodity by buying/selling futures or options contracts attempt to profit through buying and selling, based on price changes, and have no economic interest in the underlying commodity.
• Hedging is a technique to mitigate the risk of an adverse price movement in a commodity.
• Most of the companies which are using raw materials in their production are using hedging to reduce the losses as a result of the price fluctuations in the market.
Hedge Deal: In hedge deal, by using the forecast results and historical analysis of the price of the commodity a price is defined and as soon as this pre-defined price achieved, the commodity is immediately bought from this price and hedging completed.
Stop Loss Order: Again the price of the commodity is limited; however this time limit is set over the current price level and buying the commodity cancelled whenever the price exceeds this limit. This limit is also called as security level. This technique is safer than Limit Order.
Limit Order: First the price of the commodity is limited with a pre-defined price level. This limit is set under the current price level and whenever, the price of commodity reduces under this limit, the commodity is bought. However, it can not be known whether the price will fell under this limit or not. As a result of this uncertainty, this technique is not a safe way of hedging.
Order Cancels Order (OCO): The OCO is a combination of Limit-Order and Stop-Loss Order. According to this technique, both an upper and lower level is set to the price of the commodity and hedging is done whenever the price is between this range. OCO is the most common and safest technique of commodity hedging.
Factor Models and Hedging

- One of the most useful things about factor models is the additivity property of factor betas.

- Pure factor portfolios are those with a sensitivity (or factor-loading) of 1 for a given factor and of 0 for all other factors. For example:

  \[ \tilde{r}_{1,PFP} = a + 1 \times \tilde{F}_1 + 0 \times \tilde{F}_2 + \ldots + 0 \times \tilde{F}_K \]

- Consider the following general K-factor model:

  \[ \tilde{r}_i = a + \beta_1 \times \tilde{F}_1 + \beta_2 \times \tilde{F}_2 + \ldots + \beta_K \times \tilde{F}_K + \tilde{\varepsilon}_i \]
Hedging with Regression

- Regression provides a shortcut for estimating a hedge ratio that minimizes risk exposure.

- The beta coefficient in the model below is also the hedge ratio of the regression:

  \[
  \tilde{C}_i = \beta \tilde{P}_{Hedge} + \tilde{\epsilon}_i
  \]

  where
  \[
  \beta = \frac{\text{cov}(\tilde{C}_i, \tilde{P}_{hedge})}{\text{var}(\tilde{P}_{hedge})}
  \]

- \(C_i\) and \(P_{\text{hedge}}\) represent the random amount of the cash flow to be hedged and the random value of the financial security per unit sold (or bought) to be used to hedge the random amount of the cash flow, respectively.
Hedging with Regression

For a random cash flow position and a short position in the financial instrument is:

\[ \text{Var}[\tilde{C}, (-\beta \cdot \tilde{P})] = \text{var}(\tilde{C}) + \beta^2 \cdot \text{var}(\tilde{P}) - 2 \cdot \beta \cdot \text{cov}(\tilde{C}, \tilde{P}) \]

Taking the derivative of the left-hand side with respect to \( P \) and setting the derivative equal to zero gives the beta equation in the previous slide.

Note that the beta that gives the minimum variance hedge is the regression coefficient.

This suggests that the minimum variance hedge ratio can be found by using historical data to estimate regression coefficients.
Hedging with Regression

- Hedging with regression allows for cross hedging.

  - e.g., using orange juice futures (or whatever commodity one selects as the financial security for hedging) to hedge the value of lemon crop).

- Basis risk is automatically taken into account in regression hedging.

- Hedging under multiple regression models improves the hedge ratio
Credit Risk

- Credit risk arises from the possibility that borrowers and counterparties in derivative transactions may default.

Source for slides 15: Hull 2006
Credit Ratings

- In the S&P rating system, AAA is the best rating. After that comes AA, A, BBB, BB, B, CCC, CC, and C.
- The corresponding Moody’s ratings are Aaa, Aa, A, Baa, Ba, B,Caa, Ca, and C.
- Bonds with ratings of BBB (or Baa) and above are considered to be “investment grade”.
Historical Data

Historical data provided by rating agencies are also used to estimate the probability of default

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.11</td>
<td>0.25</td>
<td>0.49</td>
</tr>
<tr>
<td>Aa</td>
<td>0.02</td>
<td>0.06</td>
<td>0.09</td>
<td>0.16</td>
<td>0.23</td>
<td>0.38</td>
<td>0.54</td>
</tr>
<tr>
<td>A</td>
<td>0.05</td>
<td>0.17</td>
<td>0.34</td>
<td>0.52</td>
<td>0.72</td>
<td>1.18</td>
<td>2.05</td>
</tr>
<tr>
<td>Baa</td>
<td>0.18</td>
<td>0.49</td>
<td>0.91</td>
<td>1.40</td>
<td>1.93</td>
<td>2.99</td>
<td>4.85</td>
</tr>
<tr>
<td>Ba</td>
<td>1.17</td>
<td>3.19</td>
<td>5.58</td>
<td>8.12</td>
<td>10.39</td>
<td>14.32</td>
<td>19.96</td>
</tr>
<tr>
<td>B</td>
<td>4.55</td>
<td>10.43</td>
<td>16.19</td>
<td>21.26</td>
<td>25.90</td>
<td>34.47</td>
<td>44.37</td>
</tr>
<tr>
<td>Caa-C</td>
<td>17.72</td>
<td>29.38</td>
<td>38.68</td>
<td>46.09</td>
<td>52.29</td>
<td>59.77</td>
<td>71.38</td>
</tr>
</tbody>
</table>

Cumulative Ave Defautl Rates (%)(1970-2009, Moody’s)
The table shows the probability of default for companies starting with a particular credit rating.

A company with an initial credit rating of Baa has a probability of 0.176% of defaulting by the end of the first year, 0.494% by the end of the second year, and so on.
Do Default Probabilities Increase with Time?

- For a company that starts with a good credit rating default probabilities tend to increase with time
- For a company that starts with a poor credit rating default probabilities tend to decrease with time
Transition Matrix

<table>
<thead>
<tr>
<th>Initial rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>90.81</td>
<td>8.33</td>
<td>0.68</td>
<td>0.06</td>
<td>0.12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AA</td>
<td>0.70</td>
<td>90.65</td>
<td>7.79</td>
<td>0.64</td>
<td>0.06</td>
<td>0.14</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>0.09</td>
<td>2.27</td>
<td>91.05</td>
<td>5.52</td>
<td>0.74</td>
<td>0.26</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.95</td>
<td>86.93</td>
<td>5.30</td>
<td>1.17</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td>80.53</td>
<td>8.84</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>6.48</td>
<td>83.46</td>
<td>4.07</td>
<td>5.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0.22</td>
<td>0</td>
<td>0.22</td>
<td>1.30</td>
<td>2.38</td>
<td>11.24</td>
<td>64.86</td>
<td>19.79</td>
</tr>
</tbody>
</table>

*Source: Standard & Poor’s Credit Week (15 April 96)*
Hazard Rates vs Unconditional Default Probabilities

- The hazard rate (also called default intensity) is the probability of default for a certain time period conditional on no earlier default.
- The unconditional default probability is the probability of default for a certain time period as seen at time zero.
- What are the default intensities and unconditional default probabilities for a Caa rated company in the third year?
Hazard Rates vs Unconditional Default Probabilities

\[ \lambda_t = \lim_{h \to 0} \frac{P(t < \tau < t + h/\tau > t)}{h} \]
Hazard Rate

- The hazard rate that is usually quoted is an instantaneous rate
- If $V(t)$ is the probability of a company surviving to time $t$

$$V(t + \Delta t) - V(t) = -\lambda(t)V(t)$$

This leads to

$$V(t) = e^{-\int_0^t \lambda(t) dt}$$

The cumulative probability of default by time $t$ is

$$Q(t) = 1 - e^{-\bar{\lambda}(t)t}$$
Recovery Rate

- The recovery rate for a bond is usually defined as the price of the bond immediately after default as a percent of its face value.
- Recovery rates tend to decrease as default rates increase.
## Recovery Rates; Moody’s: 1982 to 2009

<table>
<thead>
<tr>
<th>Class</th>
<th>Mean(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st lien bank loan</td>
<td>65.6</td>
</tr>
<tr>
<td>2nd lien bank loan</td>
<td>32.8</td>
</tr>
<tr>
<td>Sen Unsec. bank loan</td>
<td>48.7</td>
</tr>
<tr>
<td>Senior Secured</td>
<td>49.8</td>
</tr>
<tr>
<td>Senior Unsecured</td>
<td>36.6</td>
</tr>
<tr>
<td>Senior Subordinated</td>
<td>30.7</td>
</tr>
<tr>
<td>Subordinated</td>
<td>31.3</td>
</tr>
<tr>
<td>Junior Subordinated</td>
<td>24.7</td>
</tr>
</tbody>
</table>
Estimating Default Probabilities

- Alternatives:
  - Use Bond Prices
  - Use CDS spreads
  - Use Historical Data
  - Use Merton’s Model
Using Bond Prices

Average default intensity over life of bond is approximately

\[
\frac{s}{1 - R}
\]

where \( s \) is the spread of the bond’s yield over the risk-free rate and \( R \) is the recovery rate.
More Exact Calculation

- Assume that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding).
- Price of risk-free bond is 104.09; price of corporate bond is 95.34; expected loss from defaults is 8.75.
- Suppose that the probability of default is $Q$ per year and that defaults always happen half way through a year (immediately before a coupon payment).
Calculations

<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Def Prob</th>
<th>Recovery Amount</th>
<th>Risk-free Value</th>
<th>Loss given Default</th>
<th>Discount Factor</th>
<th>PV of Exp Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Q</td>
<td>40</td>
<td>106.73</td>
<td>66.73</td>
<td>0.9753</td>
<td>65.08Q</td>
</tr>
<tr>
<td>1.5</td>
<td>Q</td>
<td>40</td>
<td>105.97</td>
<td>65.97</td>
<td>0.9277</td>
<td>61.20Q</td>
</tr>
<tr>
<td>2.5</td>
<td>Q</td>
<td>40</td>
<td>105.17</td>
<td>65.17</td>
<td>0.8825</td>
<td>57.52Q</td>
</tr>
<tr>
<td>3.5</td>
<td>Q</td>
<td>40</td>
<td>104.34</td>
<td>64.34</td>
<td>0.8395</td>
<td>54.01Q</td>
</tr>
<tr>
<td>4.5</td>
<td>Q</td>
<td>40</td>
<td>103.46</td>
<td>63.46</td>
<td>0.7985</td>
<td>50.67Q</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>288.48Q</td>
</tr>
</tbody>
</table>
Calculations continued

- We set $288.48Q = 8.75$ to get $Q = 3.03\%$
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution
The Risk-Free Rate

- The risk-free rate when default probabilities are estimated is usually assumed to be the LIBOR/swap zero rate (or sometimes 10 bps below the LIBOR/swap rate)
- Asset swaps provide a direct estimates of the spread of bond yields over swap rates
Real World vs Risk-Neutral Default Probabilities

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities.
- The default probabilities backed out of historical data are real-world default probabilities.
A Comparison

- Calculate 7-year default intensities from the Moody’s data, 1970-2009, (These are real world default probabilities)
- Use Merrill Lynch data to estimate average 7-year default intensities from bond prices, 1996 to 2007 (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis points
### Data from Moody’s and Merrill Lynch

<table>
<thead>
<tr>
<th>Rating</th>
<th>Default Probability (%)</th>
<th>Bond Yield Spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.245%</td>
<td>35.74</td>
</tr>
<tr>
<td>Aa</td>
<td>0.384%</td>
<td>43.67</td>
</tr>
<tr>
<td>A</td>
<td>1.179%</td>
<td>58.68</td>
</tr>
<tr>
<td>Baa</td>
<td>2.996%</td>
<td>127.53</td>
</tr>
<tr>
<td>Ba</td>
<td>14.318%</td>
<td>280.28</td>
</tr>
<tr>
<td>B</td>
<td>34.473%</td>
<td>481.04</td>
</tr>
<tr>
<td>Caa</td>
<td>59.771%</td>
<td>1103.70</td>
</tr>
</tbody>
</table>

*The benchmark risk-free rate for calculating spreads is assumed to be the swap rate minus 10 basis points. Bonds are corporate bonds with a life of approximately 7 years.
## Real World vs. Risk Neutral Hazard Rates

<table>
<thead>
<tr>
<th>Rating</th>
<th>Historical hazard rate&lt;sup&gt;1&lt;/sup&gt; % per annum</th>
<th>Hazard rate from bond prices&lt;sup&gt;2&lt;/sup&gt; (% per annum)</th>
<th>Ratio</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.04</td>
<td>0.60</td>
<td>17.0</td>
<td>0.56</td>
</tr>
<tr>
<td>Aa</td>
<td>0.05</td>
<td>0.73</td>
<td>13.2</td>
<td>0.67</td>
</tr>
<tr>
<td>A</td>
<td>0.17</td>
<td>1.15</td>
<td>6.8</td>
<td>0.98</td>
</tr>
<tr>
<td>Baa</td>
<td>0.43</td>
<td>2.13</td>
<td>4.9</td>
<td>1.69</td>
</tr>
<tr>
<td>Ba</td>
<td>2.21</td>
<td>4.67</td>
<td>2.1</td>
<td>2.46</td>
</tr>
<tr>
<td>B</td>
<td>6.04</td>
<td>8.02</td>
<td>1.3</td>
<td>1.98</td>
</tr>
<tr>
<td>Caa</td>
<td>13.01</td>
<td>18.39</td>
<td>1.4</td>
<td>5.39</td>
</tr>
</tbody>
</table>

<sup>1</sup> Calculated as $-\frac{\ln(1-d)}{7}$ where $d$ is the Moody’s 7 yr default rate. For example, in the case of Aaa companies, $d=0.00245$ and $-\ln(0.99755)/7=0.0004$ or 4bps. For investment grade companies the historical hazard rate is approximately $d/7$.

<sup>2</sup> Calculated as $s/(1-R)$ where $s$ is the bond yield spread and $R$ is the recovery rate (assumed to be 40%).
## Average Risk Premiums Earned By Bond Traders

<table>
<thead>
<tr>
<th>Rating</th>
<th>Bond Yield Spread over Treasuries (bps)</th>
<th>Spread of risk-free rate over Treasuries (bps)</th>
<th>Spread to compensate for historical default rate (bps)</th>
<th>Extra Risk Premium (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>78</td>
<td>42</td>
<td>2</td>
<td>34</td>
</tr>
<tr>
<td>Aa</td>
<td>86</td>
<td>42</td>
<td>3</td>
<td>41</td>
</tr>
<tr>
<td>A</td>
<td>111</td>
<td>42</td>
<td>10</td>
<td>59</td>
</tr>
<tr>
<td>Baa</td>
<td>169</td>
<td>42</td>
<td>26</td>
<td>101</td>
</tr>
<tr>
<td>Ba</td>
<td>322</td>
<td>42</td>
<td>132</td>
<td>148</td>
</tr>
<tr>
<td>B</td>
<td>523</td>
<td>42</td>
<td>362</td>
<td>119</td>
</tr>
<tr>
<td>Caa</td>
<td>1146</td>
<td>42</td>
<td>781</td>
<td>323</td>
</tr>
</tbody>
</table>

1. Equals average spread of our benchmark risk-free rate over Treasuries.

2. Equals historical hazard rate times \((1-R)\) where \(R\) is the recovery rate. For example, in the case of Baa, 26bps is 0.6 times 43bps.
Possible Reasons for These Results
(The third reason is the most important)

- Corporate bonds are relatively illiquid
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody’s historical data
- **Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.**
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market
Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis
Using Equity Prices: Merton’s Model

- Merton’s model regards the equity as an option on the assets of the firm.
- In a simple situation the equity value is
  \[ \max(V_T - D, 0) \]
  where \( V_T \) is the value of the firm and \( D \) is the debt repayment required.
Equity vs. Assets

The Black-Scholes-Merton option pricing model enables the value of the firm’s equity today, $E_0$, to be related to the value of its assets today, $V_0$, and the volatility of its assets, $\sigma_V$

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$
Equity vs. Assets
Equity vs. Assets

Comparison of distribution of credit returns and market returns

- Typical market returns
- Typical credit returns

Losses vs. Gains
Volatilities

\[ \sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0 \]

This equation together with the option pricing relationship enables \( V_0 \) and \( \sigma_V \) to be determined from \( E_0 \) and \( \sigma_E \).
Example

- A company’s equity is $3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is $10 million and time to debt maturity is 1 year
- Solving the two equations yields $V_0 = 12.40$ and $\sigma_v = 21.23$
- The probability of default is $N(-d_2)$ or 12.7%
The Implementation of Merton’s Model

- Choose time horizon
- Calculate cumulative obligations to time horizon. This is termed by KMV the “default point”. We denote it by $D$
- Use Merton’s model to calculate a theoretical probability of default
- Use historical data or bond data to develop a one-to-one mapping of theoretical probability into either real-world or risk-neutral probability of default.