Econometric Risk Management in Finance
Session 6

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Steps in Stress Testing

- Generate Scenarios
  - Credible worst-case scenarios relevant to portfolio positions
- Revalue Portfolio
  - Marking-to-market all financial instruments under new worst-case market rates
- Summarize results
  - Expected levels of mark-to-market loss (or gain) for each stress scenario and in which business areas the losses to be concentrated
Example: Brazilian Company

We illustrate the three steps of stress testing with the following example.

You are a Brazilian consumer products company with a significant amount of unhedged USD-denominated liabilities. You are particularly concerned about the stability of the Brazilian Real (R$), because a devaluation would make USD liabilities prohibitively expensive.

Step 1: Generate scenarios

Your economist presents two potential events:

1. A significant widening of the trade deficit, which puts pressure on the local currency, interest rates, and equity market

2. A narrowing of the trade deficit, which is a positive scenario for local markets

Following are the economist’s estimates of the effect of each scenario on the local markets over 1 day:

<table>
<thead>
<tr>
<th>Stress scenarios</th>
<th>Widening trade deficit</th>
<th>Narrowing trade deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R$/US$ exchange rate</td>
<td>up to 20% devaluation</td>
<td>no move</td>
</tr>
<tr>
<td>R$ yield curve</td>
<td>interest rates up 50%</td>
<td>interest rates down 25%</td>
</tr>
<tr>
<td>BOVESPA equity index</td>
<td>15% fall</td>
<td>10% appreciation</td>
</tr>
</tbody>
</table>
Step 3: Summarize results

A devaluation scenario could result in a direct financial loss of R$24 million for the company, while a narrowing of the trade deficit could yield a financial gain of R$4 million over 1 day. Furthermore, management should assess how each scenario might affect underlying business. For example, although a devaluation might hurt domestic sales, it could make exports into other markets more competitive. Management should then discuss whether it should take action to reduce its risk. The largest potential loss comes from the unhedged USD liabilities—for example, this could be reduced through an FX forward hedge or by purchasing a put option on R$ vs. dollar.
Scenarios

a. Generate historical scenarios based on days when market moved violently

b. Introduce market shocks and moving risk factors in isolation by large amounts to gauge sensitivity to each risk factor

c. Create anticipatory scenarios in which many market factors are moved in a consistent fashion to approximate real moves of all relevant world markets

d. Set up portfolio specific stress tests, which are based on the weakness of the portfolio itself.
## External disclosures of stress tests

To provide investors with greater risk transparency, companies may provide stress scenarios and sensitivities.

### 1. Citicorp interest rate stress test

**Citicorp Earnings-at-Risk (impact on pre-tax earnings)**

<table>
<thead>
<tr>
<th>In Millions of Dollars at December 31, 1998</th>
<th>Assuming a U.S. Dollar Rate Move of Two Standard Deviations</th>
<th>Assuming a Non-U.S. Dollar Rate Move of Two Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Increase</td>
<td>Decrease</td>
</tr>
<tr>
<td>Overnight to three months</td>
<td>$(85)</td>
<td>$87</td>
</tr>
<tr>
<td>Four to six months</td>
<td>(34)</td>
<td>38</td>
</tr>
<tr>
<td>Seven to twelve months</td>
<td>(29)</td>
<td>31</td>
</tr>
<tr>
<td><strong>Total overnight to twelve months</strong></td>
<td><strong>(148)</strong></td>
<td><strong>156</strong></td>
</tr>
<tr>
<td>Year two</td>
<td>(28)</td>
<td>22</td>
</tr>
<tr>
<td>Year three</td>
<td>12</td>
<td>(22)</td>
</tr>
<tr>
<td>Year four</td>
<td>54</td>
<td>(64)</td>
</tr>
<tr>
<td>Year five</td>
<td>119</td>
<td>(152)</td>
</tr>
<tr>
<td>Effect of discounting</td>
<td>(29)</td>
<td>39</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$ (20)</strong></td>
<td><strong>$ (21)</strong></td>
</tr>
</tbody>
</table>

(1) Primarily results from Earnings-at-Risk in Thai baht, Singapore dollar and Hong Kong dollar.

(2) Total assumes a standard deviation increase or decrease for every currency, not taking into account any covariance between currencies.

In its 1998 *Annual Report*, Citicorp discloses a simple interest rate stress test, which consists of perturbing interest rates by 2 standard deviations. Although this simple analysis is not comprehensive, shareholders and analysts get a rough perspective of Citicorp’s Net Interest Earnings (NIE) sensitivity to domestic and foreign interest rates.
2. UBS Group stress scenarios

<table>
<thead>
<tr>
<th>Country</th>
<th>Foreign exchange</th>
<th>Interest rates</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Libor/Govt.</td>
<td>Price</td>
</tr>
<tr>
<td>Europe</td>
<td>+/- 10%</td>
<td>+/- 100 bps</td>
<td>+/- 15%</td>
</tr>
<tr>
<td>North America</td>
<td>+/- 5%</td>
<td>+/- 120 bps</td>
<td>+/- 15%</td>
</tr>
<tr>
<td>Japan</td>
<td>+/- 15%</td>
<td>+/- 100 bps</td>
<td>+/- 25%</td>
</tr>
<tr>
<td>Emerging markets</td>
<td>+/- 40%</td>
<td>+500/-300 bps</td>
<td>+/- 40%</td>
</tr>
</tbody>
</table>

(+) = Market appreciation.  (-) = Market depreciation.

UBS Group reveals a sample of its stress scenarios in its 1998 Annual Report. However, loss levels are not indicated.
Lehman Brothers

- One of the major prime brokers for hedge funds till its failure in 2008
  - Prime broker
    - provides his hedge-fund customers with cash management services, securities lending services and financing services
    - Routinely lend to hedge funds to support their long positions, holding those positions as collateral
    - Rapid declines in collateral value can be a potential source of loss
    - Re-hypothecation → lend securities purchased by hedge funds to other investors.
      - It generates fees for prime broker, also creates counterparty risk for hedge fund to reclaim its securities in case of failure
Lehman Brothers

- LB operated with high levels of leverage and financed itself through the issuance of short-term debt.
- Leverage and maturity transformation (use of short term liabilities to finance longer-duration assets) as in the banking system, but lightly regulated for companies like LB (shadow banking system)
Lehman Brothers (LB)

- The Oak Group: medium sized hedge fund under LB as prime broker
- It had $22 million in long positions matched with $22 million in short positions, plus $16 million in cash in margin account
- LB having all these positions lent out $22 million in Oak’s long position (re-hypothecation)
- In case of failure, Oak Group could not regain its securities or cash and became general creditor of LB.
Lehman Brothers

- Market Liquidity after its fall:
  - Expected to decline for its clients as they can not trade
  - Leading decline in asset prices and increasing expected returns

- However: Some securities lenders exposed to LB recalled their loans, forcing those borrowers to repurchase shares and putting upward pressure on prices (Bloomberg, 2008)
- Market Complacency
- Bad regulation
- Lack of transparency
  - in issuing process: difficult to determine who owned what
- Financial policy
  * high-level of leverage (asset-to-equity ratio)
  * strong reliance on short-term debt financing
2-Copula

Definition 1 (Schweizer and Sklar [1974]) A two-dimensional copula (or 2-copula) is a function $C$ with the following properties:

1. $\text{Dom} \ C = [0, 1] \times [0, 1]$;
2. $C(0, u) = C(u, 0) = 0$ and $C(u, 1) = C(1, u) = u$ for all $u$ in $[0, 1]$;
3. $C$ is 2-increasing:

$$C(v_1, v_2) - C(v_1, u_2) - C(u_1, v_2) + C(u_1, u_2) \geq 0$$

whenever $(u_1, u_2) \in [0, 1]^2$, $(v_1, v_2) \in [0, 1]^2$ such that $0 \leq u_1 \leq v_1 \leq 1$ and $0 \leq u_2 \leq v_2 \leq 1$.

$\Rightarrow$ 2-Copulas are also doubly stochastic measures on the unit square.
Properties

i) The joint probability of all outcomes is zero if the marginal probability of any outcome is zero.

ii) If one of the variables are known with marginal probability one, then the joint probability of the other outcome is the same as the probability of the remaining uncertain outcome.

iii) 2-increasing property says that the C-volume of any 2-dimensional interval is non-negative.

The first and last properties are the general properties of multivariate c.d.f’s.

→ 2-copula can be defined as an 2-dimensional cdf whose support is contained in [0,1]² and whose one dimensional margins are uniform on [0,1].
Sklar’s Canonical Representation

**Theorem 1** Let $F_1$ and $F_2$ be 2 univariate distributions. It comes that $C(F_1(x_1), F_2(x_2))$ defines a bivariate probability distribution with margins $F_1$ and $F_2$ (because the integral transforms are uniform distributions).

**Theorem 2** Let $F$ be a 2-dimensional distribution function with margins $F_1$ and $F_2$. Then $F$ has a copula representation:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2))$$

The copula $C$ is unique if the margins are continuous.
$F(x_1, x_2)$ : continuous bivariate distribution function with

$$F_1(x_1), \ F_2(x_2) \text{ and inverse functions } F_1^{-1}, \ F_2^{-1}.$$

$x_1 = F_1^{-1}(u_1) \sim F_1$ and $x_2 = F_2^{-1}(u_2) \sim F_2 \text{ where } u_1, u_2 \text{ are uniformly dist. Variates.}$

The transforms of uniform variates are distributed as $F_i(i = 1, 2) \text{ and } 0 \leq F_i(x_i) \leq 1.$

$$F(x_1, x_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2))$$

$$= Pr[U_1 \leq u_1, U_2 \leq u_2]$$

$$= C(u_1, u_2) \text{ is the unique copula that defines } F.$$

If $x \sim F,$ and $F$ is continuous then $$(F_1(x_1), \ F_2(x_2)) \sim C,$$

and if $U \sim C,$ then $(F_1^{-1}(u_1), \ F_2^{-1}(u_2)) \sim F$
Measure of Dependence

Correlation coefficient

\[ \rho = -1 \Rightarrow \text{"the random variables are completely negatively correlated".} \]

\[ \rho = 0 \Rightarrow \text{"the random variables are uncorrelated".} \]

\[ \rho = 1 \Rightarrow \text{"the random variables are completely correlated".} \]
Sklar’s theorem

For any bivariate distribution function \((x, y)\), let \(H(x) = F(x, \infty)\) and \(G(y) = F(\infty, y)\) be the univariate marginal probability distribution functions. Then there exists a copula \(C\) such that

\[
F(x, y) = C(H(x), G(y))
\]

\(C\) is unique if the margins are continuous, and otherwise it can be uniquely determined. Therefore, the copulas are often viewed as dependence functions.

This theorem first appeared in (Sklar, 1959). The name “copula” was chosen to emphasize the manner in which a copula “couples” a joint distribution function to its univariate margins. (Nelsen, 2006)
Let $X_1$ and $X_2$ two random variables with distributions $F_1$ and $F_2$.

$$\rho (X_1, X_2) = \frac{\mathbb{E}[X_1 X_2] - \mathbb{E}[X_1] \mathbb{E}[X_2]}{\sigma [X_1] \sigma [X_2]}$$

$\rho$ is also called the Pearson correlation or the **Linear** correlation.

Using works of Tchen [1980] on superadditive functions, we can show the following results:

- If the copula of $(X_1, X_2)$ is $C^\perp$, $\rho (X_1, X_2) = 0$;
- $\rho$ is increasing with respect to the concordance order
  $$C_1 \succ C_2 \Rightarrow \rho_1 (X_1, X_2) \geq \rho_2 (X_1, X_2)$$
- $\rho (X_1, X_2)$ is bounded
  $$\rho^- (X_1, X_2) \leq \rho (X_1, X_2) \leq \rho^+ (X_1, X_2)$$
  and the bounds are attained for the Fréchet copulas $C^-$ and $C^+$. 
\[ \rho^- (X_1, X_2) = \rho^+ (X_1, X_2) = \frac{\mathbb{E}[f_1(X)f_2(X)] - \mathbb{E}[f_1(X)]\mathbb{E}[f_2(X)]}{\sigma[f_1(X)]\sigma[f_2(X)]} \]

The solution of the equation \( \rho^- (X_1, X_2) = -1 \) (or \( \rho^+ (X_1, X_2) = 1 \)) is \( f_1(x) = a_1x + b \) and \( f_2(x) = a_2x + b \) with \( a_1a_2 < 0 \) (\( a_1a_2 > 0 \) for \( \rho^+ = 1 \)).

⇒ If \( X_1 \) and \( X_2 \) are not gaussians, there exists very few solutions. For example, if \( X_1 \) and \( X_2 \) are two log-normal random variables, \( \rho^- = -1 \) can not be reached and \( \rho^+ = 1 \) if and only if \( \sigma_1 = \sigma_2 \).

\( C^- \), \( C^\perp \) and \( C^+ \) are the most important diversification function (in the sense of the correlation). Moreover, we note that

\[ C \succ C^\perp \Rightarrow \rho(X_1, X_2) \geq 0 \]

\[ C \prec C^\perp \Rightarrow \rho(X_1, X_2) \leq 0 \]
The implications of the Sklar's Theorem on practice

Copulas can be used to express a multivariate distributions in terms of its marginal distributions.

Copulas allow researchers to piece together joint distributions when only marginal distributions are known with certainty.

Another advantage of Copulas is that the marginal distributions may come from different families.
Copula with different marginals

Durleman et al., 2001
MULTi-dimensional copulas

- For an $m$-variate function $F$,

  - $C: [0,1]^m \rightarrow [0,1]$
  - $F(y_1, \cdots, y_m) = C(F_1(y_1), \cdots, F_m(y_m); \theta)$

- where $\theta$ is a parameter measures dependence between the marginals.
The Gaussian Copula

\[ C(u_1, u_2; \rho) = \phi(\phi^{-1}(u_1), \phi^{-1}(u_2); \rho) \]

\[ = \int_{-\infty}^{\phi^{-1}(u_1)} \int_{-\infty}^{\phi^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)} \left\{ -\frac{(s^2-2\rho st+t^2)}{2(1-\rho^2)} \right\} ds \, dt \]

where \( \phi \) is the c.d.f. of the standard normal distribution
\( \phi(u_1, u_2) \) is the standard bivariate normal distribution with correlation parameter \( \rho \).

The Gaussian copula is flexible in that it allows for equal degrees of positive and negative dependence.
Disadvantage: It does not have a stochastic process representation
It does not have lack of memory property
Elliptical copula surface have high density distributed at origin and \((1,1)\)

Gaussian copula with zero density near the ‘x’ and ‘y’ axis but increase as it goes away from the axis

(Seth & Myers 2007)
The Normal copula is the dependence function of the **gaussian** random vector \((X_1, X_2)\) with correlation parameter \(\rho\):

\[
C_\rho(u_1, u_2) = \Phi \left( \Phi^{-1}(u_1), \Phi^{-1}(u_2) \right)
\]

Because Normal copula is also the copula of log-normal random vector, it is the most used in finance.

The Normal copula satisfies

\[
C^- = C_{-1} < C_{\rho<0} < C_0 = C_\perp < C_{\rho>0} < C_1 = C^+
\]

It is a **comprehensive** copula (\(C^-, C_\perp\) and \(C^+\) are special cases of the Normal copula).
• **Gaussian Copula** with correlation \( \rho = 0.5 \)

Durleman et al., 2001
Student's t-Copula

Derived from Student's t multivariate distribution. This is an example of a copula with two dependent parameters is that for the bivariate t-distribution with $\nu = (\theta_1, \theta_2)$ degrees of freedom and correlation $\rho$,

$$C(u_1, u_2; \rho; \theta_1, \theta_2) = \int_{-\infty}^{t^{-1}\theta_1(u_1)} \int_{-\infty}^{t^{-1}\theta_2(u_2)} \frac{1}{2\pi(1-\theta_2^2)^{1/2}}$$

$$\times \left\{1 + \frac{(s^2 - 2\rho st + t^2)}{(n)(1-\theta_2^2)}\right\}^{-(\theta_1+2)/2} \, dsdt$$

where $t^{-1}_{\theta_i}(u_i) :$ inverse of the cdf of the standard univariate t-dist. ($\theta_i$ d.f.), $i=1,2$.

- $\theta_1$ controls the heaviness of the tails.
  - $\theta_1 < 3$ the variance does not exist
  - $\theta_1 < 5$ the fourth moment does not exist.
Archimedian Copula

- Archimedian copula surface have strong density near the origin.

(Seth & Myers 2007)
Clayton copulas

- \( C(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta} \)
- with dependence parameter \( \theta \) restricted on the region \((0, \infty)\).
- The Clayton copula cannot account for negative dependence.
- It has been used to study correlated risks because it exhibits strong left tail dependence and relatively weak right tail dependence.
Frank Copula (1979)

\[ C(u_1, u_2; \theta) = -\theta^{-1} \log \left\{ 1 + \frac{(e^{-\theta u_1-1})(e^{-\theta u_2-1})}{e^{-\theta}-1} \right\}; \quad \theta \in (-\infty, \infty) \]

• Permits negative dependence between the marginals.
• Dependence is symmetric in both tails, similar to the Gaussian and Student-t copulas.
• It can be used to model outcomes with strong positive or negative dependence.
GUMBEL Copula (1960)

- \( C(u_1, u_2; \theta) = \exp\left(-\left(\tilde{u}_1^\theta + \tilde{u}_2^\theta\right)^{1/\theta}\right) \)

- where \( \tilde{u}_j = -\log u_j \).

- The dependence parameter is restricted to the interval \([1, \infty)\).

-- does not allow negative dependence,
- exhibits strong right tail dependence and relatively weak left tail dependence.

If the outcomes are known to be strongly correlated at high values but less correlated at low values, then the Gumbel copula is an appropriate choice.

Durleman et al., 2001
Singular copulas do not have a density
\[ \partial_{1,2} C(u_1, u_2) = 0 \]

- Singular copulas with prescribed support
- Ordinal sums of singular copulas
- Shuffles of Min

The mass distribution for a shuffle of Min can be obtained by:
1. placing the mass for \( C^+ \) on \([0,1]^2\),
2. cutting \([0,1]^2\) vertically into a finite number of strips,
3. shuffling the strips with perhaps some of them flipped around their vertical axes of symmetry,
4. reassembling them to form the square again.

The resulting mass distribution will correspond to a copula called a shuffle of Min (Mikusiński, Sherwood and Taylor [1992]).
Using the \textit{duality} theorem of Frank and Schweizer [1979], it comes that if $C_- = C^-$ and $L$ is the operation $+$, we have

$$G_{\vee}^{(-1)} (u) = \inf_{\max(u_1 + u_2 - 1, 0) = u} F_1^{(-1)} (u_1) + F_2^{(-1)} (u_2)$$

and

$$G_{\wedge}^{(-1)} (u) = \sup_{\min(u_1 + u_2, 1) = u} F_1^{(-1)} (u_1) + F_2^{(-1)} (u_2)$$

We recall that $\text{VaR}_\alpha (X) = F^{-1} (\alpha)$. The corresponding dependency bounds are then

$$G_{\wedge}^{(-1)} (\alpha) \leq \text{VaR}_\alpha (X_1 + X_2) \leq G_{\vee}^{(-1)} (\alpha)$$

Numerical algorithms to compute the dependency bounds exist (for example the \textit{uniform quantisation} method of Williamson [1989]).
Dependency bounds for VaR with Gamma margins

Durleman et al., 2001
Diversification Effect

If we define the diversification effect as follows

\[ D = \frac{\text{VaR}_\alpha (X_1) + \text{VaR}_\alpha (X_2) - \text{VaR}_\alpha (X_1 + X_2)}{\text{VaR}_\alpha (X_1) + \text{VaR}_\alpha (X_2)} \]

there are situations where \( \text{VaR}_\alpha (X_1 + X_2) > \text{VaR}_\alpha (X_1) + \text{VaR}_\alpha (X_2) \).

A more appropriate definition is then

\[ \bar{D} = \frac{G_{\gamma}^{(-1)}(\alpha) - \text{VaR}_\alpha (X_1 + X_2)}{G_{\gamma}^{(-1)}(\alpha)} \]

Embrechts, McNeil and Straumann [1999] interpret \( \chi \left( C_{\gamma}^{(\alpha)}, C^+; \alpha \right) \)

\[ \chi \left( C_{\gamma}^{(\alpha)}, C^+; \alpha \right) = \frac{G_{\gamma}^{(-1)}(\alpha) - \text{VaR}_\alpha (X_1) + \text{VaR}_\alpha (X_2)}{G_{\gamma}^{(-1)}(\alpha)} \]

as “the amount by which VaR fails to be subadditive”.
LME example of Durrleman, Nickeghbali and Roncalli [2000]:

<table>
<thead>
<tr>
<th></th>
<th>AL</th>
<th>AL-15</th>
<th>CU</th>
<th>NI</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1)</td>
<td>5</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_2)</td>
<td>5</td>
<td>2</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Analytical VaR</th>
<th>Historical VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1)</td>
<td>363.05</td>
<td>445.74</td>
</tr>
<tr>
<td>(P_2)</td>
<td>1026.03</td>
<td>1274.64</td>
</tr>
</tbody>
</table>

Here are the values of \(G_{\sqrt{\cdot}}^{(-1)}(\alpha)\) for \(\alpha\) equal to 99%:

<table>
<thead>
<tr>
<th></th>
<th>(P_1)</th>
<th>(P_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical VaR</td>
<td>Historical VaR</td>
</tr>
<tr>
<td>(P_2)</td>
<td>1507.85</td>
<td>1680.77</td>
</tr>
<tr>
<td>(P_2)</td>
<td>1930.70</td>
<td>2103.67</td>
</tr>
</tbody>
</table>
Dependency bounds for VaR with LME example

Durleman et al., 2001
Credit Metrics is a tool for assessing portfolio risk and is used widely to find Value at Risk (VaR) of a portfolio. Impact of correlations among portfolios on VaR can be studied by using copulas (Seth & Myers, 2007).

(Seth & Myers 2007)
Case Study

- A study by S. Kestel, C. Saldıroglu, 2012

- Aim: To compare Value at Risk calculated by the Copula based Monte Carlo Simulation (CMC) and Standard Monte Carlo Simulation (SMC).

- The data set consists of daily prices of four currencies, which are EUR, USD, GBP and CHF, from 02.09.2009 to 02.09.2011.
- The series do not distribute normal.
- MCS under normality to calculate VaR inefficient.
- The Copula approach is used to capture the dependency structure.
Methodology

The Student’s t-copula is preferred as it is very useful in many risk management applications and is easy to implement to generate Monte Carlo scenarios (Romano, 2005).

The currency risk is the one of the fundamental risks, which is the focus of the study. We calculate Value at Risk for equity risk by using daily stock price data with both SMC and CMC methods.
Steps in the analysis

The steps below were followed during the applications and the calculations are made in MATLAB:

i) Log-returns are calculated from daily prices data.
ii) $t$ – Copula parameters are estimated by MLE.
iii) Scenarios are simulated by using Monte-Carlo with regarding to the $t$-Copula parameters (Jaworski, 2007).
iv) Value at Risk of the portfolio is calculated.
v) Backtesting the model in order to compare the performance of both methods is done.
Backtest Results

The VaR of the portfolio is calculated day by day for one year period to compare performances of SMC and CMC.
The hypothetical equally-weighted 4 sets of portfolios having the total initial value of 4000 TL reaches to a value more than 5000 TL within a year.

- The value of the portfolio increases by time
- The return would be approximately 25%.
The volatility of the portfolio moves as the Standard Approach (VaR calculated by MCS and Normality) slightly follows the portfolio volatility, 

Copula Approach (VaR calculated by MCS, t-Copula) captures the volatility movement. The number of exceeding points (indicates the days the real loss of the portfolio exceed the calculated VaR):

- The Standard Approach has 6 exceeding points,
- Copula Approach has only 2 points.

The number of exceeding points is vital for the regulation authority to approve the model.
Example of Use of Gaussian Copula

Suppose that we wish to simulate the defaults for $n$ companies. For each company the cumulative probabilities of default during the next 1, 2, 3, 4, and 5 years are 1%, 3%, 6%, 10%, and 15%, respectively.

Source: Hull 2006
Use of Gaussian Copula continued

- We sample from a multivariate normal distribution to get the $x_i$
- Critical values of $x_i$ are
  \[
  N^{-1}(0.01) = -2.33, \quad N^{-1}(0.03) = -1.88, \\
  N^{-1}(0.06) = -1.55, \quad N^{-1}(0.10) = -1.28, \\
  N^{-1}(0.15) = -1.04
  \]
Use of Gaussian Copula continued

- When sample for a company is less than -2.33, the company defaults in the first year.
- When sample is between -2.33 and -1.88, the company defaults in the second year.
- When sample is between -1.88 and -1.55, the company defaults in the third year.
- When sample is between -1.55 and -1.28, the company defaults in the fourth year.
- When sample is between -1.28 and -1.04, the company defaults during the fifth year.
- When sample is greater than -1.04, there is no default during the first five years.
A One-Factor Model for the Correlation Structure

\[ x_i = a_i F + \sqrt{1-a_i^2} Z_i \]

- The correlation between \( x_i \) and \( x_j \) is \( a_i a_j \)
- The \( i \)th company defaults by time \( T \) when \( x_i < N^{-1}[Q_i(T)] \) or

\[ Z_i < \frac{N^{-1}[Q_i(T) - a_i F]}{\sqrt{1-a_i^2}} \]

- Conditional on \( F \) the probability of this is

\[ Q_i(T|F) = N \left\{ \frac{N^{-1}[Q_i(T)] - a_i F}{\sqrt{1-a_i^2}} \right\} \]
Bank for International Settlements

- Established on 17 May 1930,
- World's oldest international financial organisation.
- Has 60 member central banks, representing countries from around the world that together make up about 95% of world GDP.
- The mission:
  - to serve central banks in their pursuit of monetary and financial stability,
  - to foster international cooperation in those areas and to act as a bank for central banks.
- The BIS pursues its mission by:
  - fostering discussion and facilitating collaboration among central banks;
  - supporting dialogue with other authorities that are responsible for promoting financial stability;
  - carrying out research and policy analysis on issues of relevance for monetary and financial stability;
  - acting as a prime counterparty for central banks in their financial transactions; and
  - serving as an agent or trustee in connection with international financial operations.
- The customers of the BIS are central banks and international organisations.

Source: www.bis.org
Basel I, II and III

- The basic goal is to provide financial stability in the world.
- Capital regulations under Basel I, (December 1992)
- Basel Committee on Banking Supervision (BCBS) → Basel Accords.
- The Basel Accords are recommendations on banking laws and regulations.
  - Internationally active banks from the G10 countries to hold a minimum total capital to absorb losses without causing systemic problems
- Basel I Accords basically focused on the credit risk.
- Basel II to remove deficiencies and to measure the risks with more sensitive methods (June 2004)
- Basel II mostly focused on the risk management
- The risks that banks are exposed can be divided into three main groups:
  - Market risk,
  - Credit risk and
  - Operational risk. With Basel II banks measure their operational risk and allocate capital for this risk.
Basel II and III

- The recent financial crisis underlined a number of weak areas in the Basel II rules → Basel III (Nov. 2010)
- Its aim is to prepare the banking industry for future economic downturn as well.
- The basic structure of Basel II remains unchanged with three mutually reinforcing pillars, but Basel III strengthens the pillars, especially Pillar 1 with enhanced minimum capital and liquidity capita
Basel II and III

- **BASEL II**
  - Pillar 1) Minimum Capital Requirement
  - Pillar 2) Supervisory Review Process
  - Pillar 3) Disclosure and Market Discipline

- **BASEL III**
  - Pillar 1) Enhanced Minimum Capital and Liquidity Requirements
  - Pillar 3) Enhanced Risk Disclosure and Market Discipline
Basel III

- The quality, consistency, and transparency of the capital base is raised and the risk coverage of the capital framework will be strengthened.
- In addition to the higher capital requirements and increased capital ratios, Basel III introduces new liquidity and leverage ratios.
- Moreover, a leverage ratio as a supplementary measure to the Basel II risk-based framework and it is introducing a series of measures to promote the build up of capital buffers in good times that can be drawn upon in periods of stress.
- A global minimum liquidity standard for internationally active banks that includes a 30-day liquidity coverage ratio requirement underpinned by a longer-term structural liquidity ratio called the Net Stable Funding Ratio. [Batra V. et al. 2010]
Basel II

- Structure of the regulation
  - Pillar I: minimum capital requirements
  - Pillar II: supervisory review
  - Pillar III: Market discipline
- General requirement for banks to hold capital equivalent to at least 8% of their risk weighted assets
Basel II
Minimum Capital requirements for

- Credit risk
  - Standardized approach
  - Internal Ratings-based approach
- Market risk
  - VaR
- Operational risk
  - Advanced Measurement Approach

Risk-weighted assets (RWA) =
Capital requirements of Market Risk and operational risk \( \times \frac{1}{0.08} \) + sum of risk-weighted assets for credit risk
## Basel II and III comparison

<table>
<thead>
<tr>
<th>Basel II</th>
<th>Requirements</th>
<th>Basel III</th>
</tr>
</thead>
<tbody>
<tr>
<td>8percent</td>
<td>Minimum Ratio of Total Capital to RWAs</td>
<td>10.5percent</td>
</tr>
<tr>
<td>2percent</td>
<td>Minimum Ratio of Common Equity to RWAs</td>
<td>4.5to7 percent</td>
</tr>
<tr>
<td>4percent</td>
<td>Tier 1 Capital to RWAs</td>
<td>6percent</td>
</tr>
<tr>
<td>2percent</td>
<td>Core Tier 1 Capital to RWAs</td>
<td>5percent</td>
</tr>
<tr>
<td>None</td>
<td>Capital Conservation Buffer to RWAs</td>
<td>2.50percent</td>
</tr>
<tr>
<td>None</td>
<td>Leverage Ratio</td>
<td>3percent</td>
</tr>
<tr>
<td>None</td>
<td>Countercyclical Buffer</td>
<td>0to2.5 percent</td>
</tr>
<tr>
<td>None</td>
<td>Minimum Liquidity Coverage Ratio</td>
<td>TDB(2015)</td>
</tr>
<tr>
<td>None</td>
<td>Minimum Net Stable Funding Ratio</td>
<td>TDB(2018)</td>
</tr>
<tr>
<td>None</td>
<td>Systemically Important Financial Institutions Charge</td>
<td>TDB(2011)</td>
</tr>
</tbody>
</table>

Source: Bank for International Settlements, Basel Committee on Banking Supervision