Econometric Risk Management in Finance
Session 1

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BASIC CONCEPTS

- **RISK** Uncertainty concerning the occurrence of a loss

- Objective Risk
- Subjective Risk
BASIC CONCEPTS

Objective Risk
The relative variation of actual loss from expected loss

Objective risk (OR) declines as the number of exposures increases. It varies by the reciprocal of the square root of the number of cases under observation.
Example: Suppose
10,000 houses having objective risk of fire
OR=10, the variation of OR is about
\[ \sqrt{10,000} = 100 \]
1,000,000 houses having objective risk of fire
OR=1, the variation of OR is about
\[ \sqrt{1,000,000} = 1000 \]
Nr. of houses increases 100 times, OR decreases 10 times
BASIC CONCEPTS

- **Subjective Risk**: Uncertainty based on a person’s mental condition or state of mind.
- High subjective risk results in conservative, prudent risk
- Less subjective risk results in less conservative behavior.
BASIC CONCEPTS

- **Chance of Loss**: Probability that an event will occur
- **Objective probability**: Long run relative frequency of an event
- Assumptions:
  - no change in the underlying conditions
  - Infinite number of observations
BASIC CONCEPTS

- **Subjective probability**: individual’s personal estimate of the chance of loss
- **PERIL**: Cause of loss
  i.e. Fire, theft, earthquakes, tornadoes
- **HAZARD**: a condition that creates or increases the chance of loss
  Three types: Physical, Moral, Morale Hazards
Physical Hazard: a physical condition that creates or increases the chance of loss

Moral Hazard: dishonesty or character defects in an individual that increase the frequency or severity of the loss.

Morale Hazard: carelessness or indifference to a loss
BASIC CATEGORIES OF RISK

- Pure and Speculative Risk
- Fundamental and Particular Risk
**BASIC CATEGORIES OF RISK**

- **Pure Risk**: there are only possibilities of loss or no loss.
- **Speculative Risk**: either profit or loss is possible.
- **Fundamental Risk**: effects entire economy or large number of persons or groups within the economy i.e. high inflation, cyclical unemployment, war etc.
BASIC CATEGORIES OF RISK

Types of Pure Risk
1. Personal Risks
2. Property Risks
3. Liability Risks
BASIC CATEGORIES OF RISK

Pure Risk

1. Personal Risk
   Risk of premature death
   Risk of insufficient income during retirement
   Risk of poor health
   Risk of Unemployment
BASIC CATEGORIES OF RISK

Pure Risk

2. Property Risks
   – Direct Loss
   – Indirect Loss

3. Liability risks
Burden of risk on society

- The size of an emergency fund must be increased
- Society is deprived of needed goods and services
- Worry and fear are present
METHODS OF HANDLING RISK

- Avoidance
- Loss Control
  - Loss prevention
  - Loss reduction
- Retention
  - Active retention
  - Passive retention
- Noninsurance Transfers
  - Transfer by contracts
  - Hedging price risks
  - Incorporation of a business firm
- Insurance
Basic Statistics and Law of Large Numbers

- Probability
  Long run frequency of the event

  with the assumption of infinite number of trials and no changes in the underlying conditions.
Basic Statistics

- **Probability distribution** is used to list events and their corresponding probabilities.
- Characterized by central tendency and dispersion like: Mean, median, standard deviation and coefficient of variation (cov).
Basic Statistics

- Mean \( \mu = \sum x_i \cdot p(x_i) \)
- Variance \( \sigma^2 = \sum (x - \mu)^2 \)
- Standard deviation \( \sigma = \sqrt{\sigma^2} \)
- Coefficient of variation c.o.v. = \((\sigma/\mu) \times 100\)
## Basic Statistics

### Example

<table>
<thead>
<tr>
<th>Amount of loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>360</td>
<td>0.50</td>
</tr>
<tr>
<td>600</td>
<td>0.20</td>
</tr>
</tbody>
</table>

\[
\mu = 300 \quad \sigma = 216.33 \quad \text{c.o.v.} = \%72
\]
## Basic Statistics

<table>
<thead>
<tr>
<th>Amount of loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>0.40</td>
</tr>
<tr>
<td>350</td>
<td>0.60</td>
</tr>
</tbody>
</table>

$$\mu = 300 \quad \sigma = 61.24 \quad \text{c.o.v.} = 20.4\%$$

- Higher the standard deviation relative to the mean, higher the risk.
Regardless of the population distribution, the distribution of sample means $\bar{X}$ will approach the normal distribution as the sample size increases.
### LAW of LARGE NUMBERS

**Example:**

<table>
<thead>
<tr>
<th>n</th>
<th>Standard error of sample mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>110.67</td>
</tr>
<tr>
<td>100</td>
<td>35.00</td>
</tr>
<tr>
<td>1,000</td>
<td>11.07</td>
</tr>
<tr>
<td>10,000</td>
<td>3.50</td>
</tr>
</tbody>
</table>
Risk Management

A systematic process for

- identification and evaluation of pure loss exposures,
- selection and implementation of the most appropriate techniques
Objectives

Preloss Objectives
- Goals of economy
- Reduction of anxiety
- Meeting externally imposed obligations

Postloss Objectives
- Survival of the firm
- Continued operation
- Stability of earnings
- Continued growth
- Social responsibility
Evaluating Potential Losses

**Loss Frequency:** Probable number of losses that may occur during some given time period

**Loss Severity:** probable size of the losses that may occur

**Maximum Possible Loss:** the worst loss that could possibly happen to the firm during its lifetime

**Maximum Probable Loss:** the worst loss that is likely to happen.
Retention the firm retains the part or all of the losses that result from a given loss exposure.

Methods:
1. Firm determines the maximum uninsured loss it can absorb without adversely effecting the firm’s earnings. Rough Rule: Max. Retention is set to 5% of annual earnings before tax
2. Max. Retention as the percentage of firm’s net working capital, such as between 1–5%.
## RM Matrix

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Severity</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Low</td>
<td>Retention</td>
</tr>
<tr>
<td>High</td>
<td>Low</td>
<td>Loss Control, Retention</td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>Insurance</td>
</tr>
<tr>
<td>High</td>
<td>High</td>
<td>Avoidance</td>
</tr>
</tbody>
</table>
Risk management process

Risk Management Processes

Strategic Risk Management
- Risk Policy
- Organizational Arrangements

Systematic Risk Identification
- Identifying risk sources and risk factors

Risk Response Planning and Risk Controlling / Monitoring:
- Handling Risk Events
- Monitoring actions against risks

Risk Assessment
- Assessing risks in terms of severity of impact and probability of occurrence

- Risk Communication and Risk Documentation
Strategic Risk Management

- Formulation of risk management objectives
- Selection of risk management objectives
- Assuring a common binding declaration of aim of risk management working together with the company management

Systematic Risk Identification

- Determination of what, how and why things happen and the objective is to find the types of risks the company might be exposed to
- Definition of the manageable risks
  - Preparation of contingency and business continuity plans in order to handle the risks which are outside the control of the company
- Risk Identification Techniques:
  - Checklists
  - Brainstorming Sessions
  - Diagramming Techniques
  - Regular Meetings
  - Event Tree Analysis (ETA)
  - Failure Mode and Effects Analysis (FMEA)
Risk Assessment

- Analyzing the available information in order to find out how often those specified risks might occur and the magnitude of the outcomes
- Prioritization of risks and determination of how much the risk could be tolerable or not.
- Developing solution strategies to the defined and prioritized risks
  - Qualitative Risk Impact Analysis
    - Scenario Building
- Extrapolative approach
- Normative approach
  - Likelihood–Impact Matrix
- Quantitative Risk Impact Analysis
  - Expected Value Analysis
  - Benefit–Cost Analysis
  - Monte Carlo Simulation
Risk Response Planning and Risk Controlling/Monitoring

- Prepare people for actions so that they can react quickly and follow plans that were previously prepared
- Making second scenarios to mitigate the risks
- Developing solution strategies to the defined and prioritized risks
- Risk Response Planning
  - Risk Avoidance
  - Risk Mitigation
  - Risk Transfer
  - Risk Acceptance
- Risk Controlling and Planning
  - Status Reports
  - Issue Logs

- Risk Communication and Risk Documentation
Case Study

RISK ANALYSIS OF PROCUREMENT ACTIVITIES FOR COPPER COMMODITIES
Buzdogan and Kestel, 2009
Relationship between the copper consumption and price
What is the importance of Risk Management in Procurement Activities?

Procurement chain and possible risk areas

The importance of Risk Management in global procurement:
- Change in the market condition (technology, production techniques, services, suppliers, customers, etc.)
- The globalization in the market and increasing competition
- Worldwide competition of external capital
Lack of a systematic and detailed risk management in procurement activities
Highly variable market and supplier structure relating with many commodities
Dependency to the supplier
Economic crisis

- Developing a detailed and structured risk management in the procurement activities.
  - Current problems that we are facing with copper suppliers
  - Price volatility in the raw copper market

- Risk management in the procurement activities of copper suppliers and economic effects

Solution Procedure of the Practical Part
The price of the copper in the fiscal year 2007/2008 according to LME (London Metal Exchange)

The average hedged copper price of the fiscal year 2007/2008

The range of the hedged copper price in the fiscal year 2007/2008

→ Hedging reached its goal of securing the budget and mitigating the risk of facing with higher copper prices.
Hedging & the power of hedging at Siemens IA CD GP


- Although hedging mitigate the risk of facing with higher copper prices, it could not take the advantages of the reduced copper prices.
  - Not fixing the hedging technique and criteria in the beginning of the each FY.
  - Increasing the data & forecast analysis.
  - Using a more flexible hedging strategy (e.g. making the hedging decisions of each month one month before).

Models for copper prices and hedging prices

\[ \Delta^1 y_t = -1.185 + 1.019 \Delta^1 y_{t-1} - 0.512 \Delta^1 y_{t-2} + \varepsilon_1 + 1.687 \varepsilon_{t-1} \]

\[ \Delta^1 z_t = -6.581 - 0.829 \Delta^1 z_{t-1} + \varepsilon_t + 0.962 \varepsilon_{t-1} \]
<table>
<thead>
<tr>
<th>Months</th>
<th>Differences</th>
<th>Calculation</th>
<th>Risk Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>September-06</td>
<td>0.000</td>
<td>0</td>
<td>No Risk</td>
</tr>
<tr>
<td>October-06</td>
<td>18.048</td>
<td>18.048&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>November-06</td>
<td>-31.090</td>
<td>31.09&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>December-06</td>
<td>-71.838</td>
<td>50.317&lt;71.838&lt;100.634</td>
<td>Medium Risk</td>
</tr>
<tr>
<td>February-07</td>
<td>-122.508</td>
<td>122.508&gt;100.634</td>
<td>High Risk</td>
</tr>
<tr>
<td>January-07</td>
<td>-124.376</td>
<td>124.376&gt;100.634</td>
<td>High Risk</td>
</tr>
<tr>
<td>March-07</td>
<td>-71.289</td>
<td>50.317&lt;71.289&lt;100.634</td>
<td>Medium Risk</td>
</tr>
<tr>
<td>April-07</td>
<td>49.257</td>
<td>49.257&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>May-07</td>
<td>43.299</td>
<td>43.299&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>June-07</td>
<td>32.026</td>
<td>32.026&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>July-07</td>
<td>13.105</td>
<td>13.105&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>August-07</td>
<td>-16.774</td>
<td>16.774&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>September-07</td>
<td>-18.122</td>
<td>18.122&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>October-07</td>
<td>-18.933</td>
<td>18.933&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>November-07</td>
<td>-116.029</td>
<td>116.029&gt;100.634</td>
<td>High Risk</td>
</tr>
<tr>
<td>December-07</td>
<td>-112.875</td>
<td>112.875&gt;100.634</td>
<td>High Risk</td>
</tr>
<tr>
<td>January-08</td>
<td>-39.542</td>
<td>39.542&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>February-08</td>
<td>0.043</td>
<td>0</td>
<td>No Risk</td>
</tr>
<tr>
<td>March-08</td>
<td>7.318</td>
<td>7.318&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>April-08</td>
<td>16.227</td>
<td>16.227&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>May-08</td>
<td>2.499</td>
<td>2.499&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>June-08</td>
<td>-2.708</td>
<td>2.708&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>July-08</td>
<td>-3.032</td>
<td>3.032&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>August-08</td>
<td>-20.607</td>
<td>20.607&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>September-08</td>
<td>-36.066</td>
<td>36.066&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>October-08</td>
<td>-22.252</td>
<td>22.252&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>November-08</td>
<td>8.710</td>
<td>8.710&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>December-08</td>
<td>33.414</td>
<td>33.414&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>January-09</td>
<td>42.576</td>
<td>42.576&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>February-09</td>
<td>36.907</td>
<td>36.907&lt;50.317</td>
<td>Low Risk</td>
</tr>
<tr>
<td>March-09</td>
<td>27.456</td>
<td>27.456&lt;50.317</td>
<td>Low Risk</td>
</tr>
</tbody>
</table>
Market Situation:
- In the market, there is just a few big suppliers (7)
- The technologies (extruding, rolling) are established worldwide.
Risk Identification & Preparation of Checklist

<table>
<thead>
<tr>
<th>Strategic</th>
<th>Operations</th>
<th>Financial</th>
<th>Compliance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Dynamics</td>
<td>Customer Relationship Management</td>
<td>Market</td>
<td>Code of Conduct</td>
</tr>
<tr>
<td>Planning &amp; Resource Allocation</td>
<td>Supply Chain Management</td>
<td>Liquidity &amp; Credit</td>
<td>Legal &amp; Intellectual Property</td>
</tr>
<tr>
<td>Major Initiatives</td>
<td>Physical Assets</td>
<td>Accounting &amp; Reporting</td>
<td>Regulatory Environment</td>
</tr>
<tr>
<td>Valuation &amp; Pricing</td>
<td>Construction Contracts &amp; Project Management</td>
<td>Tax</td>
<td>Contract</td>
</tr>
<tr>
<td>Joint Ventures</td>
<td>Product Lifecycle Management</td>
<td>Capital Structure</td>
<td></td>
</tr>
<tr>
<td>Communication &amp; Investor Relations</td>
<td>Information Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Governance</td>
<td>Human Resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Empowerment &amp; Accountability</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A&D Criteria Set - Strategy FY2007

A&D Criteria Set - Supplier Evaluation

Brainstorming

Meeting

Risk Analysis Checklist

A & D Criteria Set - Risk FY2007
### CHECKLIST QUESTIONS - OPERATIONAL PART

<table>
<thead>
<tr>
<th>Sub-Risk Level</th>
<th>CHECKLIST QUESTIONS - OPERATIONAL PART</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>Does the supplier have an adequate Total Quality Management (TQM) system?</td>
<td>[ ] Yes [ ] No [ ] Don't Know</td>
</tr>
<tr>
<td>Warranties</td>
<td>Are the warranties clearly defined?</td>
<td>[ ] Yes [ ] No [ ] Don't Know</td>
</tr>
</tbody>
</table>
Determination of Risk Levels

Weights of Risk Categories

- Operational; 18%
- Financial; 38%
- Compliance; 8%
- Strategic; 30%
- General; 6%
- Procurement

Risks
Risk identification

General Weight Diagram

- Financial: 38%
- Operations: 18%
- Strategic: 30%
- General: 6%
- Compliance: 8%
### TOTAL RISK TABLE

<table>
<thead>
<tr>
<th>Risk Criterion</th>
<th>Weights</th>
<th>Supplier A</th>
<th>Supplier B</th>
<th>Supplier C</th>
<th>Supplier D</th>
<th>Supplier E</th>
<th>Supplier F</th>
<th>Supplier G</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Risk</td>
<td>6.00%</td>
<td>0.578</td>
<td>0.333</td>
<td>0.392</td>
<td>0.461</td>
<td>0.438</td>
<td>0.629</td>
<td>0.331</td>
</tr>
<tr>
<td>Strategic Risk</td>
<td>30.00%</td>
<td>0.478</td>
<td>0.472</td>
<td>0.428</td>
<td>0.397</td>
<td>0.485</td>
<td>0.503</td>
<td>0.283</td>
</tr>
<tr>
<td>Operational Risk</td>
<td>18.00%</td>
<td>0.315</td>
<td>0.373</td>
<td>0.195</td>
<td>0.318</td>
<td>0.296</td>
<td>0.340</td>
<td>0.176</td>
</tr>
<tr>
<td>Financial Risk</td>
<td>38.00%</td>
<td>0.467</td>
<td>0.474</td>
<td>0.258</td>
<td>0.395</td>
<td>0.326</td>
<td>0.290</td>
<td>0.253</td>
</tr>
<tr>
<td>Compliance Risk</td>
<td>8.00%</td>
<td>0.481</td>
<td>0.483</td>
<td>0.477</td>
<td>0.464</td>
<td>0.547</td>
<td>0.792</td>
<td>0.370</td>
</tr>
<tr>
<td>Total Risk</td>
<td>100.00%</td>
<td>0.451</td>
<td>0.448</td>
<td>0.323</td>
<td>0.391</td>
<td>0.392</td>
<td>0.423</td>
<td>0.262</td>
</tr>
</tbody>
</table>
Expected Utility Model

**Paradigm:** the *expected utility hypothesis* postulates that a decision maker takes decisions about random prospects based on $u(x)$ rather than on the actual wealth $x$, for some *utility function* $u(\cdot)$. He prefers a random loss $X$ to $Y$, starting from initial wealth $w$, if $E[u(w-X)] \geq E[u(w-Y)]$.

Utility functions should have some plausible properties, for instance, they should be increasing. Some classes of functions that may be used as utility functions are exhibited, and their properties examined.
Expected Utility Method

**Utility functions:**

- A decision maker takes decision based on a *utility function* \( u(w) \) rather than on his current capital \( w \).
- A (capitalistic) decision maker wants to maximize \( u(w) \).
• $u(w)$ is non-decreasing: $u'(w) \geq 0$. „the richer, the better“

• Gossen’s first law (the law of diminishing marginal utility) usually applies to capital:
  $u'$ decreasing $\Rightarrow u'' < 0$
  in general $u$ is concave (decreasing marginal utility)
A function $f$ is said to be convex if for all $x$ and $y$:

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y) \quad , \quad 0 \leq \lambda \leq 1$$

and concave if:

$$f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda)f(y) \quad , \quad 0 \leq \lambda \leq 1$$
Properties:

1. $f$ convex $\iff \forall x_0 \exists$ line $l(x) = a + bx$ with $l(x_0) = f(x_0)$ and $l(x) \leq f(x) \forall x$.

2. $f$ concave $\iff \forall x_0 \exists$ line $l(x) = a + bx$ with $l(x_0) = f(x_0)$ and $l(x) \geq f(x) \forall x$.

3. Every convex and concave function is continuous.

4. Every non-negative linear combination of convex(concave) functions is convex(concave).
Lineair transformations

- The utility function $u(x)$ and its linear transforms $a \cdot u(x) + b$ for $a > 0$ are equivalent:

  $$E[u(w - X)] \leq E[u(w - Y)] \text{ if and only if }$$
  $$E[a \cdot u(w - X) + b] \leq E[a \cdot u(w - Y) + b]$$

- So we can select one, e.g. by requiring $u(0) = 0$ and $u(1) = 1$
Jensen’s Inequality

Theorem 1.2.3. Jensen’s inequality

- If \( v(.) \) is convex then \( E(v(X)) \geq v(E(X)) \)
- If \( v(.) \) is concave then \( E(v(X)) \leq v(E(X)) \)

Also the reverse holds and

\( v(.) \) linea r \( \iff \) \( E(v(X)) = v(E(X)) \) for all \( X \)
Decreasing marginal utility is similar to risk-aversion:
If decision-maker has the choice between insuring and not-insuring he/she has to choose between: $E(u(w-X))$ (not-insuring) and $u(w-E(X))$ (insuring)
But: $E(u(w-X)) \leq u(E(w-X)) = u(w-E(X))$

Decision maker pays rather the fixed premium $E(X)$ than paying the risk $X$. He/she is risk-averse.
Also:
Risk-lover $\iff$ increasing marginal utility
Risk-indifferent $\iff$ constant marginal utility
Suppose that an insured has a concave utility function and that he faces a loss $X$. An insurer asks a premium $P$.

The **insured** will benefit if:

$$E(u(w-X)) \leq u(w-P)$$

$u$ non-decreasing, continuous $\rightarrow$ maximal premium $P^+$ that he is prepared to pay is that $P^+$ for which

$$E(u(w-X)) = u(w-P^+)$$
Example

With a utility function \( u(\cdot) \), we can define its absolute risk aversion coefficient \( r(w) = -u''(w)/u'(w) \)

⇒ Approximation for zero-utility premium:
solve \( \mathbb{E}[u(w-X)] = u(w-P^+) \) for \( P^+ \), after having replaced both sides by Taylor approximations:

\[
u(w-P^+) \approx u(w-\mu) + (\mu-P^+)u'(w-\mu);
\]

and

\[
u(w-X) \approx u(w-\mu) + (\mu-X)u'(w-\mu) + \frac{\mu}{2}(\mu-X)^2u''(w-\mu)
\]

⇒ \( \mathbb{E}[u(w-X)] \approx u(w-\mu) + \frac{\mu}{2}\sigma^2 u''(w-\mu) \)

⇒ \( \frac{\mu}{2}\sigma^2 u''(w-\mu) \approx (\mu-P^+)u'(w-\mu) \)

⇒ \( P^+ \approx \mu - \frac{\mu}{2}\sigma^2 \times u''(w-\mu)/u'(w-\mu) \)

⇒ \( P^+ \approx \mu + \frac{\mu}{2}\sigma^2 r(w-\mu) \)

⇒ \( P^+ \approx \text{net premium} + \frac{1}{2} \times \text{variance} \times \text{‘risk aversion’} \)
Classes of utility functions.

The following classes of utility functions have properties which make them suitable for practical use:

\[ u(w) = w \quad \text{linear utility} \]
\[ u(w) = -(\alpha - w)^2 \quad \text{quadratic utility} \quad w \leq \alpha \]
\[ u(w) = \log(\alpha + w) \quad \text{logarithmic utility} \quad w > -\alpha \]
\[ u(w) = -\alpha e^{-\alpha w} \quad \text{exponential utility} \quad \alpha > 0 \]
\[ u(w) = w^c \quad \text{power utility} \quad w > 0, \ 0 < c \leq 1 \]
Example: Exponential premium.

\[ E[u(w-X)] = u(w-P^+) \text{ with } \exp(\alpha) \text{ utility } \Rightarrow \]
\[ E[-\alpha e^{-\alpha(w-X)}] = -\alpha e^{-\alpha(w-P^+)} \text{, so } E[e^{\alpha X}] = \exp(\alpha P^+) \Rightarrow P^+ = \frac{1}{\alpha} \ln(m_X(\alpha)). \]

This premium does not depend on the initial wealth \( w \).

The exponential utility function has a constant risk aversion \( r(w) \equiv \alpha \)

so the approximate premium above has this same property.
Example Quadratic utility.

Let the utility function be \( u(x) = 10x - x^2 \) \((x<5)\);
assume the loss \( X \sim \text{Bernoulli}(\frac{1}{2}) \).

Determine \( P^+ \) as a function of the initial capital \( w, w \in [0, 5] \).

\[ \downarrow \text{exercise} \]

Expected utility after loss \( X \):

\[ E[u(w-X)] = 11w - 5\frac{1}{2} - w^2 \]

this must be equal to:

\[ u(w-P) = 10(w-P) - (w-P)^2 \]

so

\[ P = P(w) = +\sqrt{(5\frac{1}{2} - w)^2 + \frac{1}{4}} - (5 - w), \ w \in [0, 5] \]

\[ \Rightarrow P'(w) > 0 \text{ holds} \]

Also \( r'(w) > 0 \): ‘the richer, the more conservative’

\[ \Rightarrow \text{Quadratic utility does not describe the behavior of insureds very well.} \]