2. Grundbegriffe

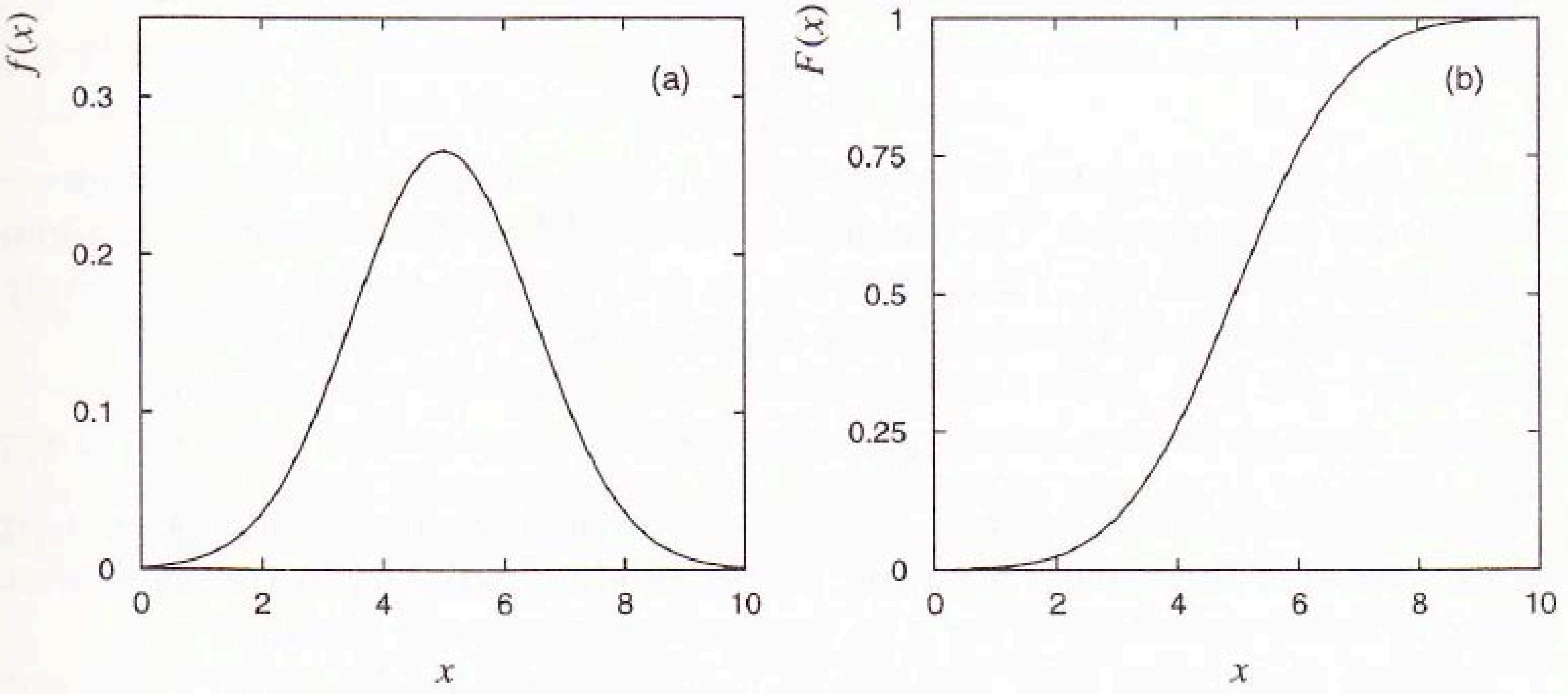

Fig. 1.3 (a) A probability density function $f(x)$. (b) The corresponding cumulative distribution function $F(x)$.

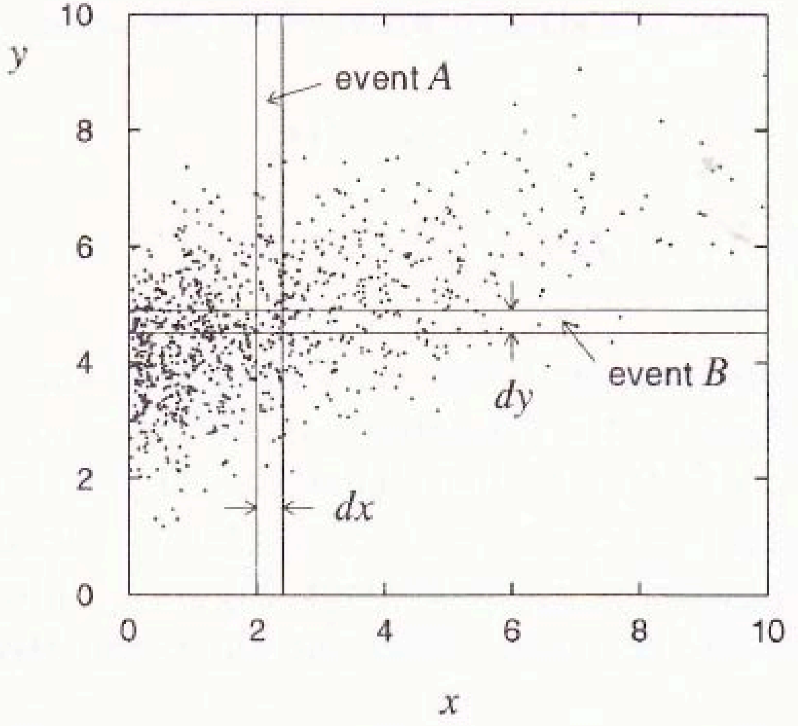

Fig. 1.4 A scatter plot of two random variables x and y based on 1000 observations. The probability for a point to be observed in the square given by the intersection of the two bands (the event $A \cap B$) is given by the joint p.d.f. times the area element, $f(x, y)dxdy$.

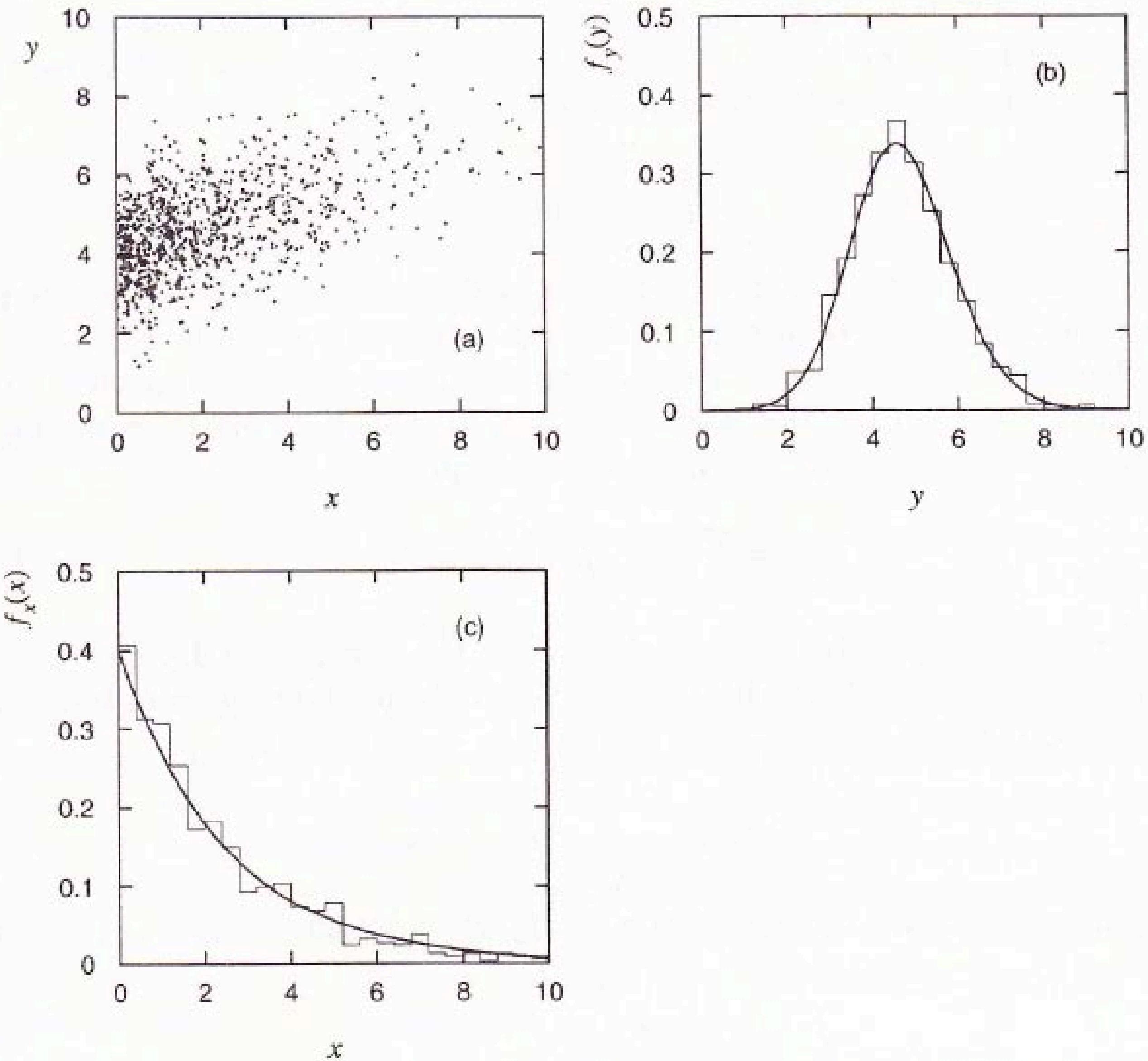

Fig. 1.5 (a) The density of points on the scatter plot is given by the joint p.d.f. $f(x, y)$. (b) Normalized histogram from projecting the points onto the y axis with the corresponding marginal p.d.f. $f_y(y)$. (c) Projection onto the x axis giving $f_x(x)$.

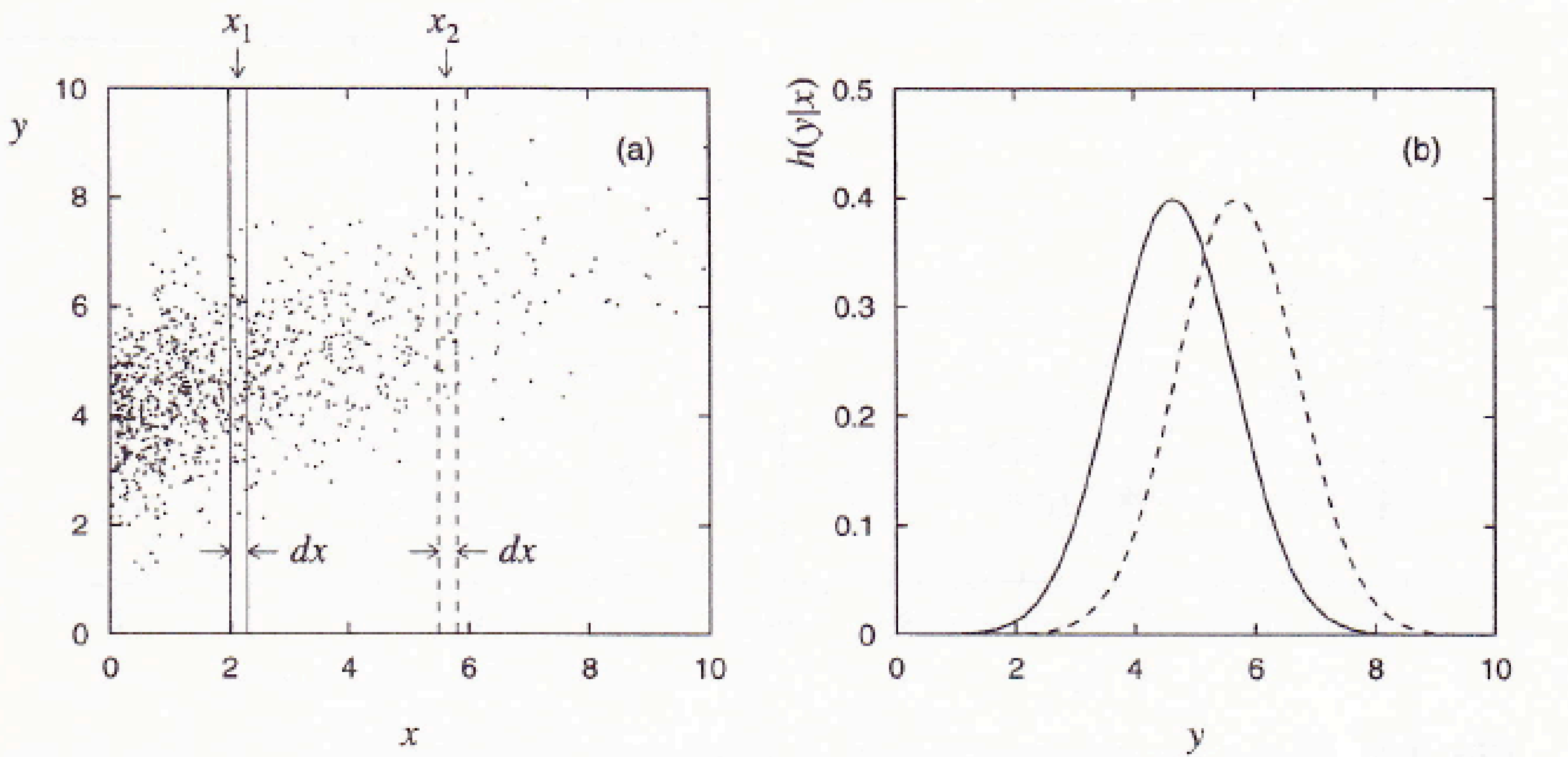

Fig. 1.6 (a) A scatter plot of random variables x and y indicating two infinitesimal bands in x of width dx at x_1 (solid band) and x_2 (dashed band). (b) The conditional p.d.f.s $h(y|x_1)$ and $h(y|x_2)$ corresponding to the projections of the bands onto the y axis.

Verallgemeinerung auf n Variablen

Variablen: $x_1, x_2, x_3, \ldots\ldots, x_n$

Verteilungsfkt: $F(x_1, x_2, x_3, \ldots X_n) = P(x_1 < X_1, x_2 < X_2, \ldots\ldots, x_n < X_n)$

Erwartungswerte:

$$E\{h(x_1, x_2, \ldots, x_n)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{\infty} h(x_1, \ldots x_n)\, f(x_1, \ldots, x_n)\, dx_1\, d \ldots dx$$

$$E(X_r) = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{\infty} X_r \cdot f(x_1, x_2, \ldots, x_n)\, dx_1 \cdot dx_2 \cdot \ldots \cdot dx_n$$

Variablen sind unabhängig, wenn:

$$f(x_1, x_2, \ldots, x_n) = g_1(x_1) \cdot g_2(x_2) \cdot \ldots \cdot g_n(x_n)$$

Zwischen allen Variablenpaaren (x_i, x_j) können Kovarianzen berechnet werden:

$$c_{ij} = \mathrm{cov}(x_i, x_j) = E\{(x_i - \mu_i)(x_j - \mu_j)\}$$

Kovarianzmatrix:

$$C := \begin{pmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ddots & \\ \vdots & & & \\ c_{n1} & & & c_{nn} \end{pmatrix}$$

Diagonale: $c_{ii} = \sigma^2(x_i)$

Symmetrisch: $c_{ij} = c_{ji}$

Kurzschreibweise: $\vec{x} = (x_1, x_2, \ldots x_n)$ $\quad \vec{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$

$$C = E\{(\vec{x} - \vec{\mu})(\vec{x} - \vec{\mu})^T\}$$

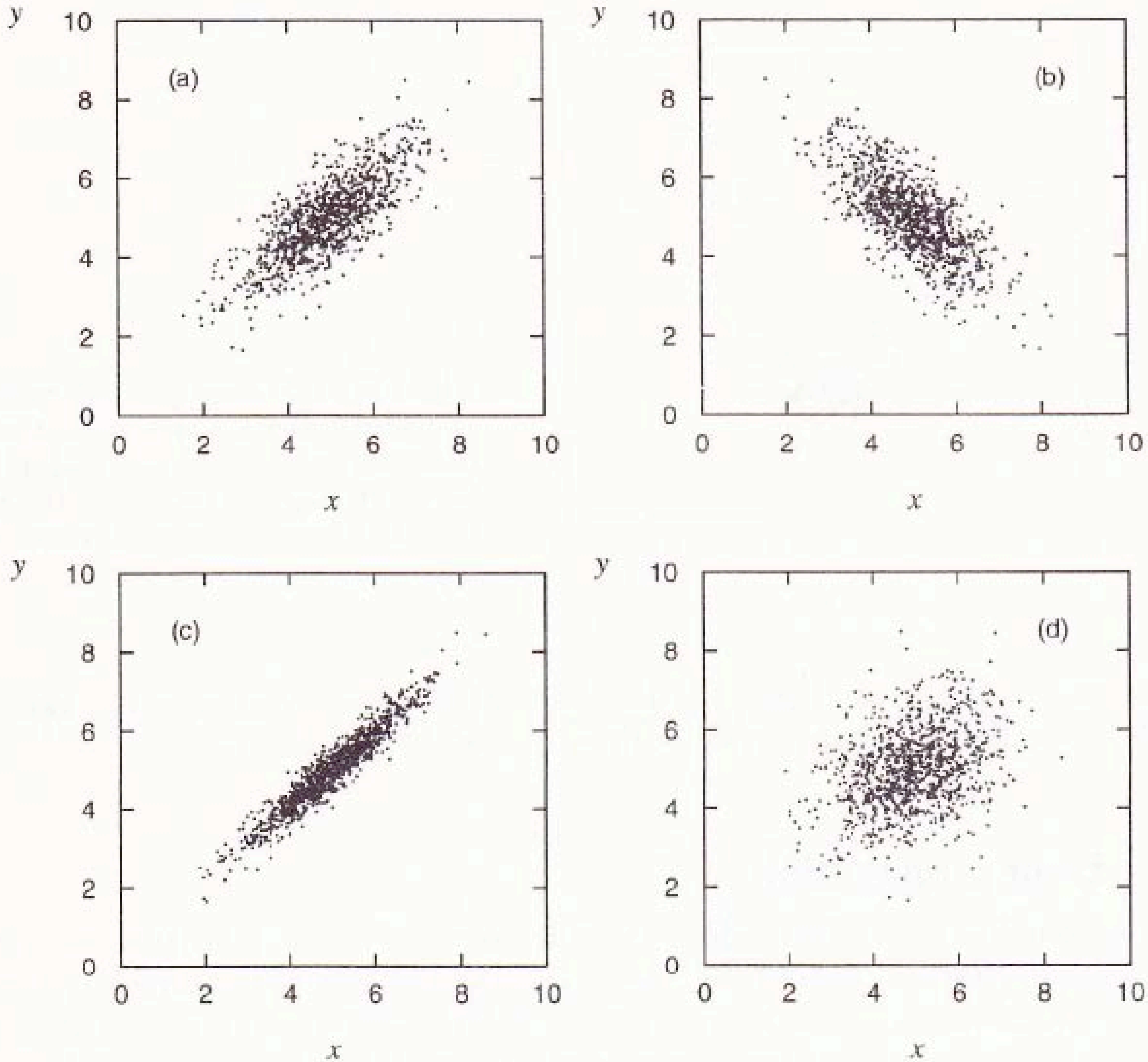

Fig. 1.9 Scatter plots of random variables x and y with (a) a positive correlation, $\rho = 0.75$, (b) a negative correlation, $\rho = -0.75$, (c) $\rho = 0.95$, and (d) $\rho = 0.25$. For all four cases the standard deviations of x and y are $\sigma_x = \sigma_y = 1$.