# Problem Set for Hadron Collider Physics 2015

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Problem Set 6

**Your solutions have to be handed in by 10:10 am on Tuesday, June 23$^{rd}$ 2015. Please drop them into the mailbox number 1 on the ground floor of the Gustav-Mie building!**

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1. **Electroweak parameters**

   Use the world's best measured values of $m_Z$ and the measurement of $\sin^2\theta_W$ from Møller scattering ($\sin^2\theta_W = 0.2397 \pm 0.0013$) to predict the $W$ boson mass. Assign an uncertainty on this prediction as well and compare this with the world's best direct measurement of $m_W$. Comment on the comparison. **[4 points]**

2. **Dirac lagrangian density and local gauge invariance**

   Consider the following lagrangian density:

$$L = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi \tag{1}$$

   (a) Obtain the Dirac equation by making use of the Euler-Lagrange equation: **[1 point]**

$$\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\bar{\psi})}\right) = \frac{\partial L}{\partial\bar{\psi}} \tag{2}$$

   (b) Show that the lagrangian density $L$ is invariant under *global* $U(1)$ transformations: **[2 points]**

$$\psi \to e^{i\alpha}\psi \tag{3}$$

   (c) Show that the $L$ is *not* invariant under *local* $U(1)$ gauge transformations: **[2 points]**

$$\psi \to e^{i\alpha(x)}\psi \tag{4}$$

   (d) Show that the modified lagrangian density obtained introducing a new vector field $A_\mu$:

$$L = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{5}$$

   (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) is invariant under local $U(1)$ gauge transformations, if $A_\mu$ transforms as follows : **[2 points]**

$$A_\mu \to A_\mu + (1/e)\partial_\mu\alpha \tag{6}$$

   (e) Show that a mass term for the vector field $m^2 A_\mu A^\mu$ would violate the gauge local invariance. **[1 point]**

   In short: the requirement of local gauge invariance "produces" the interaction term between fermions and bosons, but only for massless bosons.

3. **Higgs boson at the LHC**

A Higgs boson has been discovered at the LHC with a mass of 125 GeV. The dominant production and decay channel for this mass is $gg \to H \to b\bar{b}$. Nevertheless, this is a very difficult channel to probe. Let's try to understand why in a semi-quantitative way.

(a) What is the production cross section for a Higgs boson at the LHC at a center of mass energy of 8 TeV in this production channel? Cite your source. **[1 point]**

(b) Make a list of the main background processes and corresponding (approximate) cross sections. **[2 points]**

(c) Assuming that the cross section for producing $b\bar{b}$ in the final state via QCD production is 100 nb, and that of the Higgs boson is 1 pb, compute the statistical significance $N_s/\sqrt{N_b}$ of a signal that could be measured with a dataset corresponding to $20\,\mathrm{fb}^{-1}$ of integrated luminosity. For what integrated luminosity would one reach a statistical significance of five standard deviations? **[2 points]**

(d) Can you think of a way of "saving" the measurement of the Higgs decay into $b\bar{b}$? What can one do to still measure this channel? **[1 point]**