Each step in a discrete-time quantum walk is typically understood to have two basic components: a "coin-toss" which produces a random superposition of two states, and a displacement which moves each component of the superposition by different amounts. Here we report on the experimental implementation of a walk in momentum space with a spinor Bose-Einstein condensate (BEC) subject to a quantum ratchet realized with a pulsed, off-resonant optical lattice. By an appropriate choice of the lattice detuning, we show how the atomic momentum can be entangled with the internal spin states of the atoms. For the coin-toss, we propose to use a microwave pulse to mix these internal states. We present first experimental results of such a quantum walk based on a new type of ratchet, and through a series of simulations, demonstrate how our system can allow for the investigation of possible biases and classical-to-quantum dynamics in the presence of natural and engineered noise. Moreover, the same setup offers the possibility to realize classical random walks by applying a random sequence of intensities and phases of the time-dependent lattice chosen according to a given probability distribution. This distribution converts on average into the final momentum distribution of the atoms. In particular, it is shown that a power-law distribution for the intensities results in a classical Lévy walk in momentum space. Finally, we propose another implementation of a BEC quantum walk in reciprocal or quasimomentum space with exciting possibilities to investigate the effects of long-range quantum correlations induced by atom-atom interactions. General framework for the characterisation of complex processes with memory.